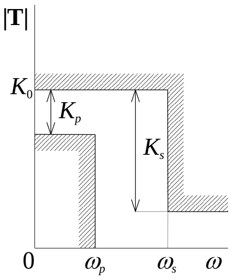
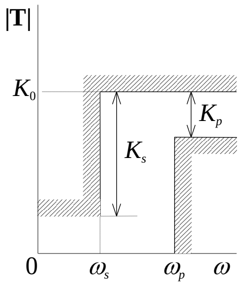
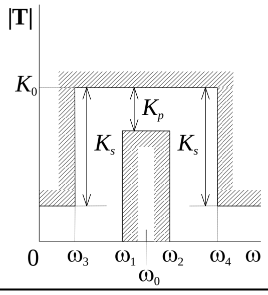
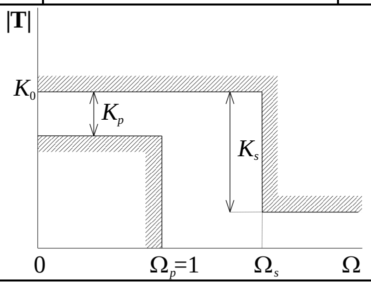
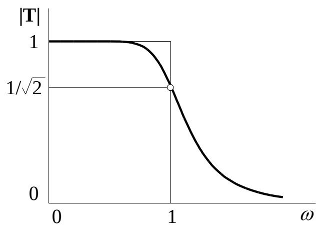
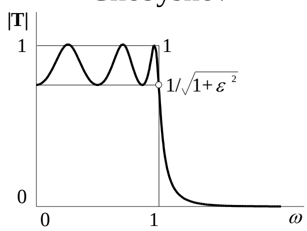
Active Analog Cascade Filter Design

Step-by-step cascade design of Butterworth and Chebyshev filters for lowpass, highpass, and bandpass responses.

Design Steps

	lowpass	highpass	bandpass
Specification			
Frequency Transformation	$\Omega_s = \frac{\omega_s}{\omega_p}$	$\Omega_s = \frac{\omega_p}{\omega_s}$	$\Omega_s = \frac{\omega_4 - \omega_3}{\omega_2 - \omega_1}$ $\omega_0^2 = \omega_1\omega_2 = \omega_3\omega_4$
Lowpass Prototype Specification			
M	$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}}$		
Response	Butterworth 	Chebyshev 	
Order	$n = \frac{\ln M}{\ln \Omega_s}$ Round up to an integer.	$n = \frac{\cosh^{-1} M}{\cosh^{-1} \Omega_s}$ Round up to an integer.	
Butterworth Pole Angles	If n is odd, $\psi_k = k\frac{180°}{n} \quad k = 0, \pm1, \pm2, \ldots, \pm(n-1)/2$ If n is even, $\psi_k = k\frac{180°}{n} \pm \frac{90°}{n} \quad k = 0, \pm1, \pm2, \ldots, \pm(n-2)/2$		
Pole Locations	$\Omega_{0p} = \frac{1}{\left(10^{0.1K_p} - 1\right)^{1/2n}}$ $\Omega_{0s} = \frac{\Omega_s}{\left(10^{0.1K_s} - 1\right)^{1/2n}}$	$\varepsilon = \sqrt{10^{0.1K_p} - 1}$ $v = \frac{1}{n}\sinh^{-1}\left(\frac{1}{\varepsilon}\right)$ $\Sigma_k = -\sinh(v)\cos(\psi_k)$ $\Omega_k = \cosh(v)\sin(\psi_k)$	

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<table>
<tr><td>Pole Locations (cont'd)</td><td colspan="2">$\Omega_0 = \sqrt{\Omega_{0p}\Omega_{0s}}$
$Q_{0LPk} = \frac{1}{2\cos\psi_k}, \psi_k \neq 0°$
If n is odd, then there is a real pole at $-\Omega_0$.</td><td>$\Omega_{0k} = \sqrt{\Sigma_k^2 + \Omega_k^2}$
$Q_{0LPk} = \frac{\Omega_{0k}}{-2\Sigma_k}, \psi_k \neq 0°$
If n is odd, then there is a real pole at $-\Omega_{00}$.</td></tr>
<tr><td>Inverse Freq. Transformation</td><td>$\omega_{0k} = \omega_p \Omega_{0k}$</td><td>$\omega_{0k} = \frac{\omega_p}{\Omega_{0k}}$</td><td>Geffe algorithm</td></tr>
<tr><td>Geffe Algorithm (bandpass only)</td><td colspan="3">real pole:
$q = \frac{\omega_0}{\omega_2 - \omega_1}$
$Q_0 = \frac{q}{\Omega_0}$

complex poles:
$D = \frac{\Omega_0}{Q_{0\mathrm{LP}} q}$, $E = 4 + \frac{\Omega_0^2}{q^2}$
$G = \sqrt{E^2 - 4D^2}$
$Q_0 = \frac{1}{D}\sqrt{\frac{1}{2}(E+G)}$
$K = \frac{DQ_0}{2}$, $W = K + \sqrt{K^2 - 1}$
$\omega_{01} = W\omega_0$, $\omega_{02} = \frac{1}{W}\omega_0$</td></tr>
<tr><td>1st order Cct</td><td>Buffered RC</td><td>Buffered RC</td><td>None required</td></tr>
<tr><td>2nd order Cct</td><td>Biquad, Sallen-Key</td><td>Biquad</td><td>Biquad, Friend</td></tr>
<tr><td>Scaling</td><td colspan="3">For each stage, $k_f = \omega_{0k}$. Choose k_m for proper element values.</td></tr>
<tr><td>Gain</td><td colspan="3">Provide gain or attenuation to meet the K_0 specification.</td></tr>
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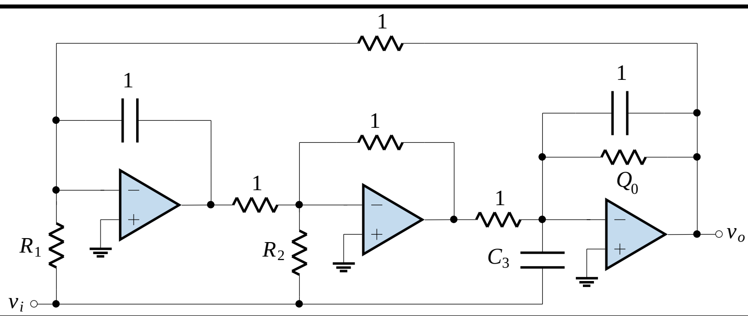
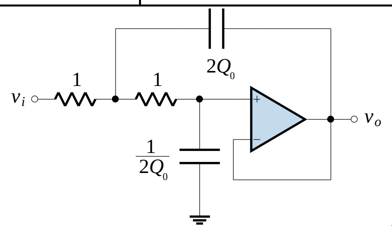
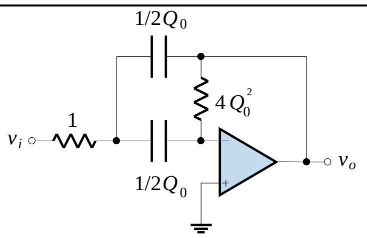
Second-order Circuits

<table>
<tr><td rowspan="2">Biquad (universal)

Design Eqns.</td><td colspan="3">
</td></tr>
<tr><td>$R_1 = 1/H$
$R_2 = \infty$
$C_3 = 0$</td><td>$R_1 = \infty$
$R_2 = \infty$
$C_3 = H$</td><td>$R_1 = \infty$
$R_2 = Q_0/H$
$C_3 = 0$</td></tr>
<tr><td>Sallen-Key (lowpass)</td><td colspan="3">

$H = 1$ is fixed</td></tr>
<tr><td>Friend (bandpass)</td><td colspan="3">

$H = -2Q_0^2$ is fixed</td></tr>
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