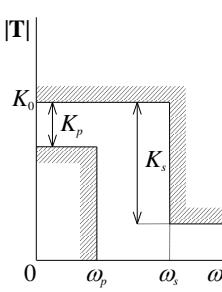
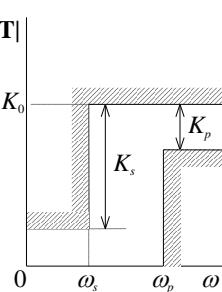
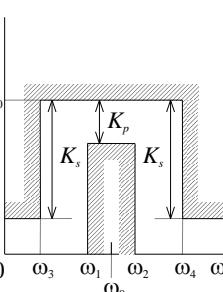
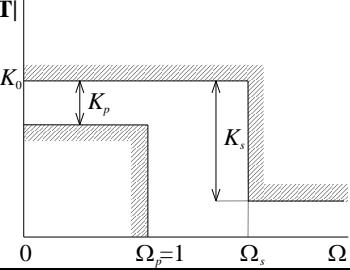
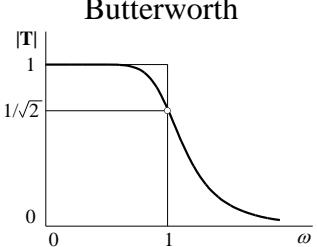
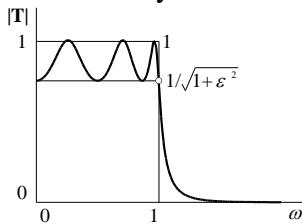


# Active Analog Cascade Filter Design

Step-by-step cascade design of Butterworth and Chebyshev filters for lowpass, highpass, and bandpass responses.

## Design Steps

	lowpass	highpass	bandpass
Specification	 <p>Graph of magnitude  T  vs frequency <math>\omega</math>. The plot shows a step response starting at <math>K_0</math> at <math>\omega = 0</math>, dropping to <math>K_p</math> at <math>\omega = \omega_p</math>, staying constant until <math>\omega = \omega_s</math>, and then dropping again to <math>K_0</math>.</p>	 <p>Graph of magnitude  T  vs frequency <math>\omega</math>. The plot shows a step response starting at <math>K_0</math> at <math>\omega = 0</math>, dropping to <math>K_s</math> at <math>\omega = \omega_s</math>, staying constant until <math>\omega = \omega_p</math>, and then dropping again to <math>K_0</math>.</p>	 <p>Graph of magnitude  T  vs frequency <math>\omega</math>. The plot shows a step response starting at <math>K_0</math> at <math>\omega = 0</math>, dropping to <math>K_s</math> at <math>\omega = \omega_3</math>, staying constant until <math>\omega = \omega_1</math>, dropping to <math>K_p</math> at <math>\omega = \omega_1</math>, staying constant until <math>\omega = \omega_2</math>, dropping to <math>K_s</math> at <math>\omega = \omega_2</math>, staying constant until <math>\omega = \omega_4</math>, and then dropping again to <math>K_0</math>.</p>
Frequency Transformation	$\Omega_s = \frac{\omega_s}{\omega_p}$	$\Omega_s = \frac{\omega_p}{\omega_s}$	$\Omega_s = \frac{\omega_4 - \omega_3}{\omega_2 - \omega_1}$ $\omega_0^2 = \omega_1\omega_2 = \omega_3\omega_4$
Lowpass Prototype Specification		 <p>Graph of magnitude  T  vs frequency <math>\Omega</math>. The plot shows a step response starting at <math>K_0</math> at <math>\Omega = 0</math>, dropping to <math>K_p</math> at <math>\Omega = \Omega_p = 1</math>, staying constant until <math>\Omega = \Omega_s</math>, and then dropping again to <math>K_0</math>.</p>	
$M$			$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}}$
Response	 <p>Graph of magnitude  T  vs frequency <math>\omega</math>. The plot shows a Butterworth response starting at 1 at <math>\omega = 0</math>, decreasing to <math>1/\sqrt{2}</math> at <math>\omega = 1</math>, and then continuing to roll off.</p>	 <p>Graph of magnitude  T  vs frequency <math>\omega</math>. The plot shows a Chebyshev response with a passband containing ripples, followed by a sharp roll-off.</p>	
Order	$n = \frac{\ln M}{\ln \Omega_s}$ Round up to an integer.	$n = \frac{\cosh^{-1} M}{\cosh^{-1} \Omega_s}$ Round up to an integer.	
Butterworth Pole Angles	If $n$ is odd, $\psi_k = k \frac{180^\circ}{n}$ $k = 0, \pm 1, \pm 2, \dots, \pm (n-1)/2$ If $n$ is even, $\psi_k = k \frac{180^\circ}{n} \pm \frac{90^\circ}{n}$ $k = 0, \pm 1, \pm 2, \dots, \pm (n-2)/2$		
Pole Locations	$\Omega_{0p} = \frac{1}{(10^{0.1K_p} - 1)^{1/2n}}$ $\Omega_{0s} = \frac{\Omega_s}{(10^{0.1K_s} - 1)^{1/2n}}$	$\varepsilon = \sqrt{10^{0.1K_p} - 1}$ $\nu = \frac{1}{n} \sinh^{-1}\left(\frac{1}{\varepsilon}\right)$ $\Sigma_k = -\sinh(\nu) \cos(\psi_k)$ $\Omega_k = \cosh(\nu) \sin(\psi_k)$	

## C.2

Pole Locations (cont'd)	$\Omega_0 = \sqrt{\Omega_{0p}\Omega_{0s}}$ $Q_{0LPk} = \frac{1}{2\cos\psi_k}, \psi_k \neq 0^\circ$ If $n$ is odd, then there is a real pole at $-\Omega_0$ .	$\Omega_{0k} = \sqrt{\Sigma_k^2 + \Omega_k^2}$ $Q_{0LPk} = \frac{\Omega_{0k}}{-2\Sigma_k}, \psi_k \neq 0^\circ$ If $n$ is odd, then there is a real pole at $-\Omega_{00}$ .
Inverse Freq. Transformation	$\omega_{0k} = \omega_p \Omega_{0k}$	$\omega_{0k} = \frac{\omega_p}{\Omega_{0k}}$ Geffe algorithm
Geffe Algorithm (bandpass only)	real pole: $q = \frac{\omega_0}{\omega_2 - \omega_1}$ $Q_0 = \frac{q}{\Omega_0}$	complex poles: $D = \frac{\Omega_0}{Q_{0LP}q}, E = 4 + \frac{\Omega_0^2}{q^2}$ $G = \sqrt{E^2 - 4D^2}$ $Q_0 = \frac{1}{D} \sqrt{\frac{1}{2}(E+G)}$ $K = \frac{DQ_0}{2}, W = K + \sqrt{K^2 - 1}$ $\omega_{01} = W\omega_0, \omega_{02} = \frac{1}{W}\omega_0$
1st order Cct	Buffered RC	Buffered RC
2nd order Cct	Biquad, Sallen-Key	Biquad
Scaling	For each stage, $k_f = \omega_{0k}$ . Choose $k_m$ for proper element values.	
Gain	Provide gain or attenuation to meet the $K_0$ specification.	

### Second-order Circuits

Biquad (universal)			
Design Eqns.	$R_1 = 1/H$ $R_2 = \infty$ $C_3 = 0$	$R_1 = \infty$ $R_2 = \infty$ $C_3 = H$	$R_1 = \infty$ $R_2 = Q_0/H$ $C_3 = 0$
Sallen-Key (lowpass)			$H = 1$ is fixed
Friend (bandpass)			$H = -2Q_0^2$ is fixed