Lecture 6B – The Transformer Principle

Transformer electric and magnetic equivalent circuits. Stray capacitance. Sign convention.

Transformer Electric and Magnetic Equivalent Circuits

A transformer can be as simple as the following arrangement:

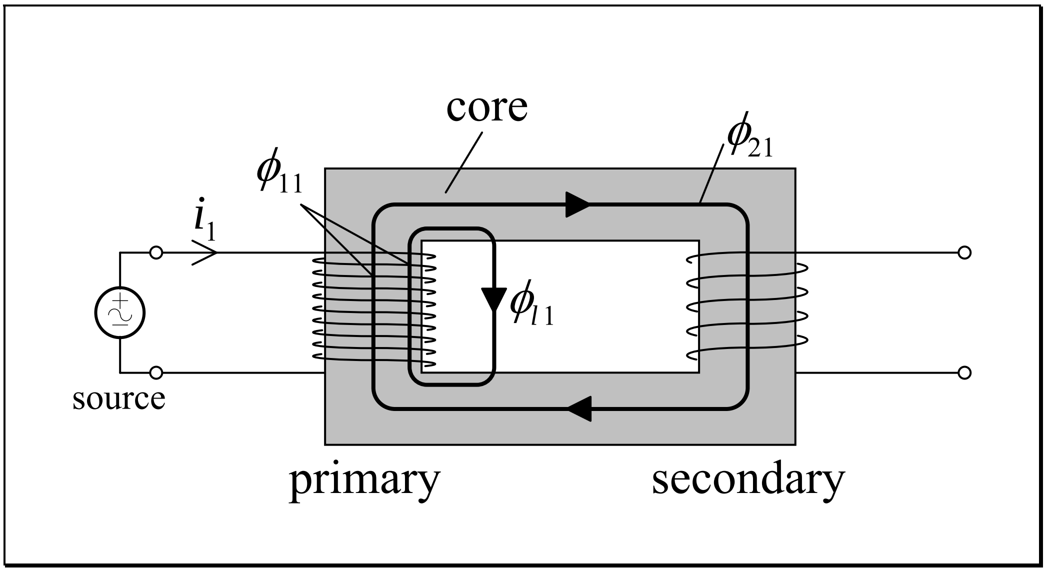


Figure 6B.1

It consists of a ferromagnetic core (to direct the flux along a particular path) and two windings. One winding is connected to a source, the other to a load. The source supplies current to the primary winding, which creates a flux in the core. Most of this flux streams through the core and links the secondary winding (ϕ_{21}). Some of the flux leaks through the air (ϕ_{l1}).

Suppose the source is a sinusoidally varying voltage. If the resistance of the winding is small, then the induced emf across the primary winding will equal the source voltage. *Show this using KVL.* Faraday's law then tells us that the flux will be sinusoidal.

Since a time varying flux is linking the secondary winding, there will be an emf induced in that winding. With no load connected to the secondary winding, this emf has no effect on the flux (the secondary is an open circuit, so there is no current). The transformer is just an iron cored inductor.

If we now connect a load to the secondary winding, the induced emf will force a sinusoidal current. The emf acts like an ordinary voltage source. Remember: an emf is not a passive voltage like that across a capacitor – there is something going on behind the scenes (chemistry for a battery, motion for a generator, Faraday's law for a transformer, etc) that allows it to supply a steady current.

The flux in the transformer, due entirely to the secondary winding carrying a current, is shown below for the case of i_2 coming out of the top secondary winding:

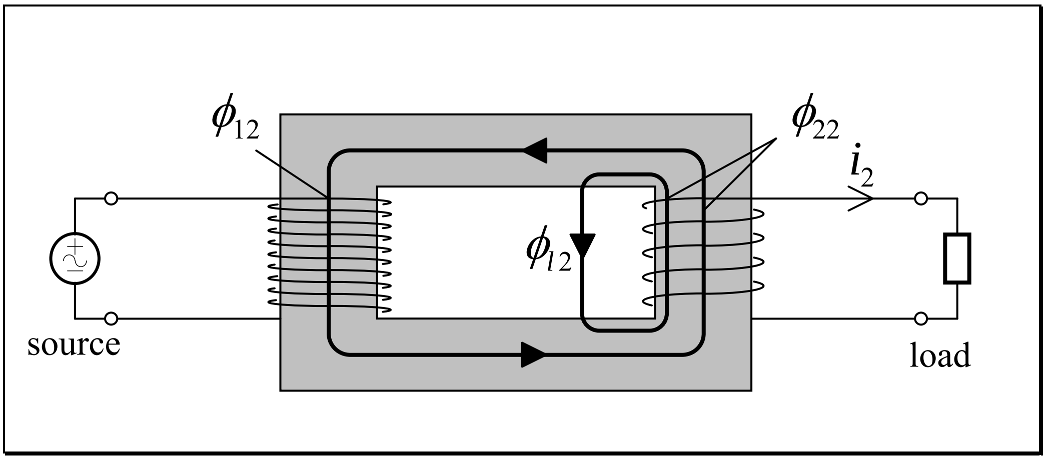


Figure 6B.2

We can now imagine the total flux streaming through the transformer:

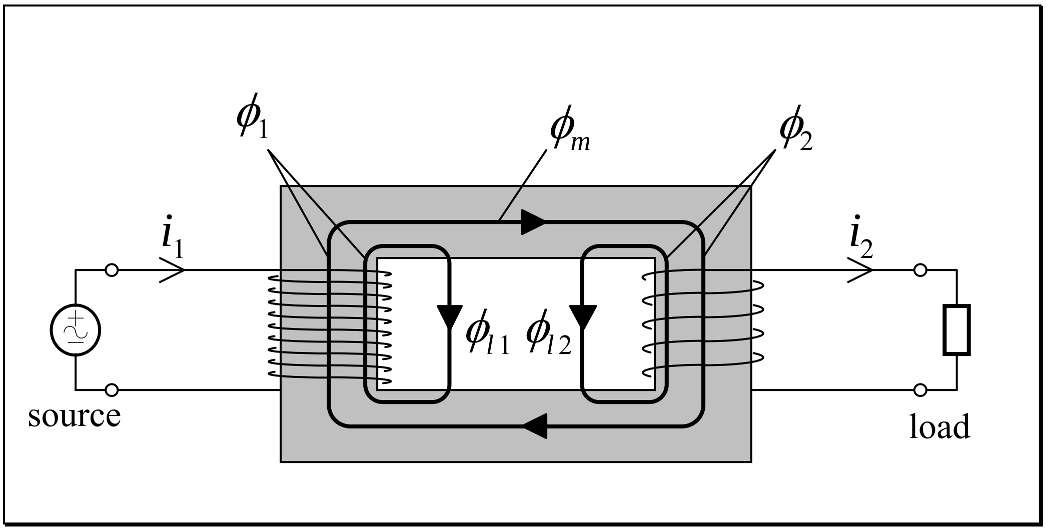


Figure 6B.3

When do the directions of current shown above apply? What polarity do the induced emfs on the windings have? Can the load current be in the other direction?

We can draw the following conclusions from the above picture:

$$\begin{aligned} \phi_m &= \phi_{21} - \phi_{12} \\ \phi_1 &= \phi_m + \phi_{l1} \\ \phi_2 &= \phi_m - \phi_{l2} \end{aligned} \tag{6B.1}$$

The flux linking the primary winding is ϕ_1. The flux linking the secondary winding is ϕ_2.

The total flux linkages of each winding are:

$$\begin{aligned} \lambda_1 &= N_1 \phi_1 \\ \lambda_2 &= N_2 \phi_2 \end{aligned} \tag{6B.2}$$

We will now apply KVL to each winding, taking into account the winding resistance and the induced emf. On the source side we have:

$$v_1 = R_1 i_1 + \frac{d\lambda_1}{dt} \tag{6B.3}$$

where v_1 is the source voltage. On the load side we have:

$$v_2 = -R_2 i_2 + \frac{d\lambda_2}{dt} \tag{6B.4}$$

where v_2 is the load voltage. *Verify that this is true and label the voltage polarities on the previous diagram. What is* R_2 *?*

The emfs induced in the two windings can also be expressed by:

$$\frac{d\lambda_1}{dt} = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_m}{dt} + N_1 \frac{d\phi_{l1}}{dt} \tag{6B.5a}$$

$$\frac{d\lambda_2}{dt} = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_m}{dt} - N_2 \frac{d\phi_{l2}}{dt} \tag{6B.5b}$$

We can now see two distinct parts of the induced emf. One is associated with the leakage flux, the other with the mutual flux.

Let's define the leakage inductance of each winding:

$$L_{l1} = \frac{N_1 \phi_{l1}}{i_1}, \qquad L_{l2} = \frac{N_2 \phi_{l2}}{i_2} \tag{6B.6}$$

and the emfs induced in the windings by the mutual flux:

$$e_1 = N_1 \frac{d\phi_m}{dt}, \qquad e_2 = N_2 \frac{d\phi_m}{dt} \tag{6B.7}$$

From the above equations we can see that:

$$\boxed{\frac{e_1}{N_1} = \frac{e_2}{N_2}} \tag{6B.8}$$

This equation comes from Faraday's law, and is called a *volts per turn balance*.

It relates the primary and secondary voltages on a transformer using the turns ratio. The KVL equations can now be written:

$$v_1 = R_1 i_1 + L_{l1} \frac{di_1}{dt} + e_1 \tag{6B.9a}$$

$$v_2 = -R_2 i_2 - L_{l2} \frac{di_2}{dt} + e_2 \tag{6B.9b}$$

Our model of the transformer can now be represented by a core that exhibits no leakage, since we can explicitly show the leakage component of inductance in the electrical circuit:

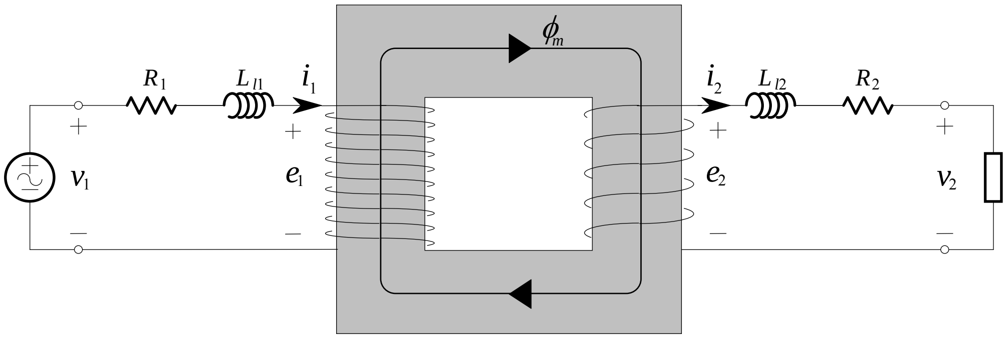


Figure 6B.4

Notice how the core has lost its leakage flux. We have transferred it so that it is represented in the electrical circuit, instead of the magnetic circuit.

Verify that Eqs. (6B.9a) *are KVL around the primary and secondary in the above model.*

The magnetic equivalent circuit of the arrangement above is therefore:

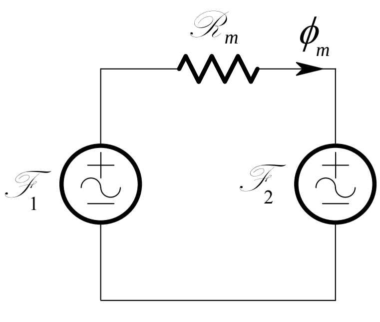


Figure 6B.5

6B.6

We can reduce the magnetic circuit further so that there is only one mmf that creates the flux:

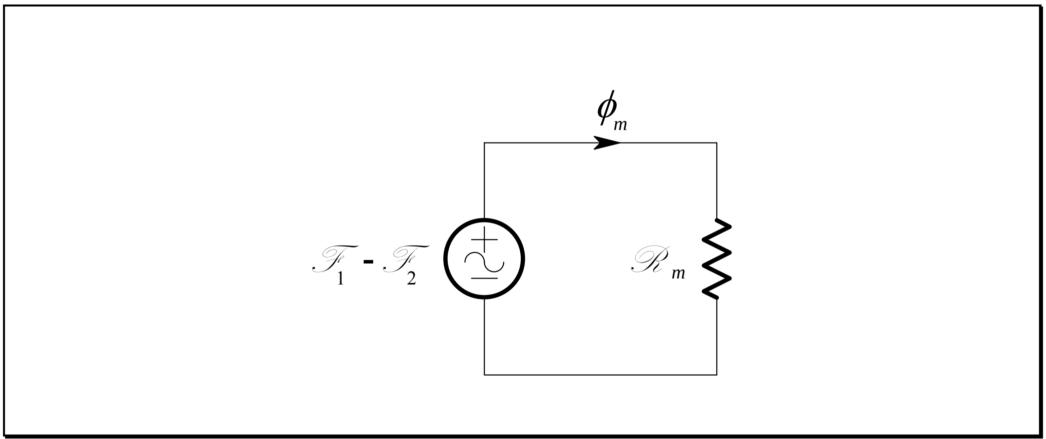


Figure 6B.6

Now imagine that this imaginary mmf was produced by the primary winding only. We can call this mmf a magnetising mmf since it is the cause of the flux in the core (it magnetises it).

The imaginary mmf is given by:

$$N_1 i_m = N_1 i_1 - N_2 i_2 \tag{6B.10}$$

where i_m is called the *magnetizing current*. This equation is called an *mmf balance*, and gives the relationship between currents in a transformer. Since we imagine a magnetizing current producing a flux, it must be in an inductive circuit (inductors are the producers of flux). The above expression can be rearranged to give:

$$\begin{aligned} i_2' &= \frac{N_2}{N_1} i_2 \\ i_1 &= i_m + i_2' \end{aligned} \tag{6B.11}$$

This is KCL at the following node:

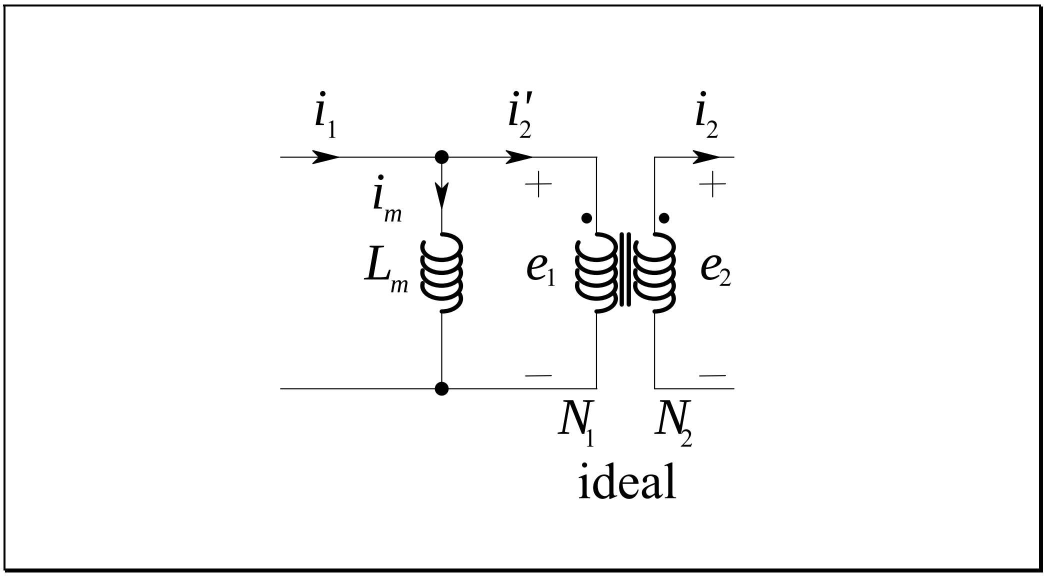


Figure 6B.7

The inductance L_m is defined as:

$$L_m = \frac{N_1 \phi_m}{i_m} \tag{6B.12}$$

Show why we define it to be this value. Hint: the inductor is in parallel with the ideal primary winding. Use Faraday's law.

Our electrical equivalent circuit for a transformer is now:

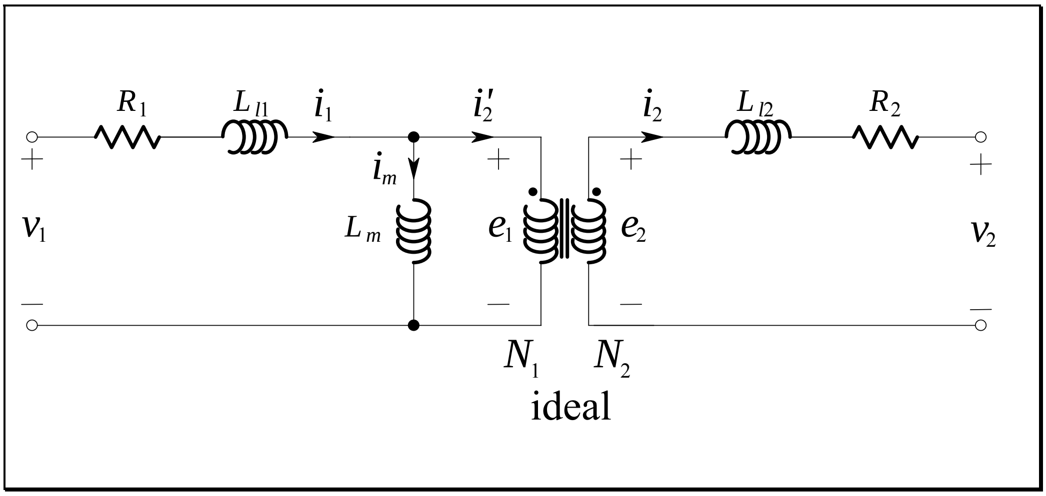


Figure 6B.8

The ideal transformer has relationships defined by Eqs. (6B.8) and (6B.11). We can "reflect" impedances from one side of the ideal transformer to the other. For example, let $\mathbf{Z}_2$ be the impedance seen by $\mathbf{E}_2$. Then the impedance seen by the primary is:

$$\mathbf{Z}_2' = \frac{\mathbf{E}_1}{\mathbf{I}_2'} = \frac{N_1/N_2}{N_2/N_1}\frac{\mathbf{E}_2}{\mathbf{I}_2} = \frac{N_1^2}{N_2^2}\mathbf{Z}_2 \tag{6B.13}$$

We can replace the ideal transformer and everything on the load side by the equivalent impedance $\mathbf{Z}_2'$.

Stray Capacitance

Stray interturn and interwinding capacitance affects the transformer's frequency response. At low frequencies (50 Hz) they are ignored.

Sign Convention

A dot is placed on the ends of the primary and secondary winding of a transformer to indicate the polarity of the winding.

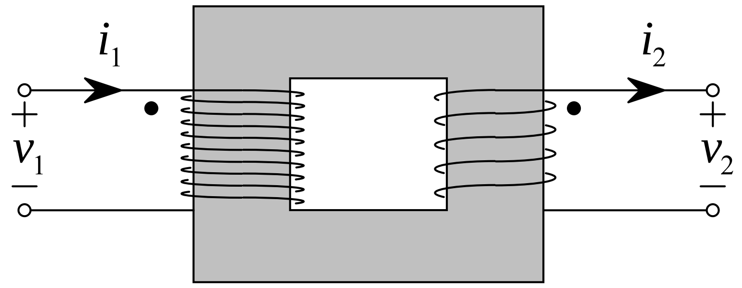


Figure 6B.9

At any instant:

- voltages on the windings have the same polarity with respect to the dot
- one winding has current into the dot, whilst the other has current out of the dot

Summary

- A simple transformer consists of two windings wrapped around a ferromagnetic core. A sinusoidal source supplies current to the *primary* winding, which produces a changing flux in the core. This changing flux induces an emf in the *secondary* winding.

- Faraday's Law applied to a transformer leads to a relation known as a "volts per turn balance": $\frac{e_1}{N_1} = \frac{e_2}{N_2}$.

- In an ideal transformer with no magnetising current, Ampère's Law leads to a relation known as an "mmf balance": $N_1 i_1 = N_2 i_2$.

- The electrical equivalent circuit for a transformer has several elements: primary winding resistance R_1, primary leakage reactance L_{l1}, primary magnetising reactance L_m, and ideal transformer, secondary winding resistance R_2 and secondary leakage reactance L_{l2} .

- Approximations to the equivalent circuit of a transformer can be used in many cases to obtain quite reasonable models of transformer behaviour (it depends on the construction of the transformer and the external circuit).

References

Slemon, G. and Straughen, A.: *Electric Machines*, Addison-Wesley Publishing Company, Inc., Sydney, 1982.

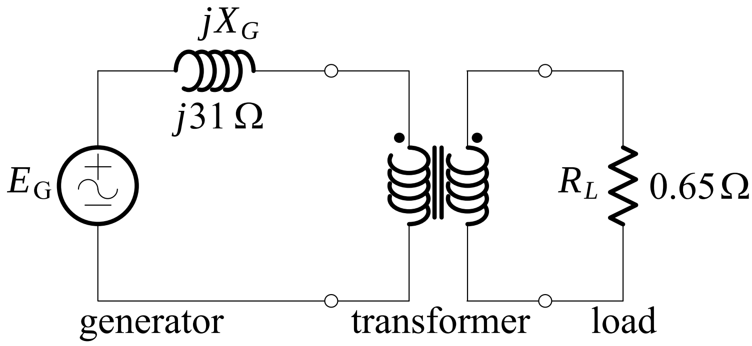
6B.10

Problems

1.

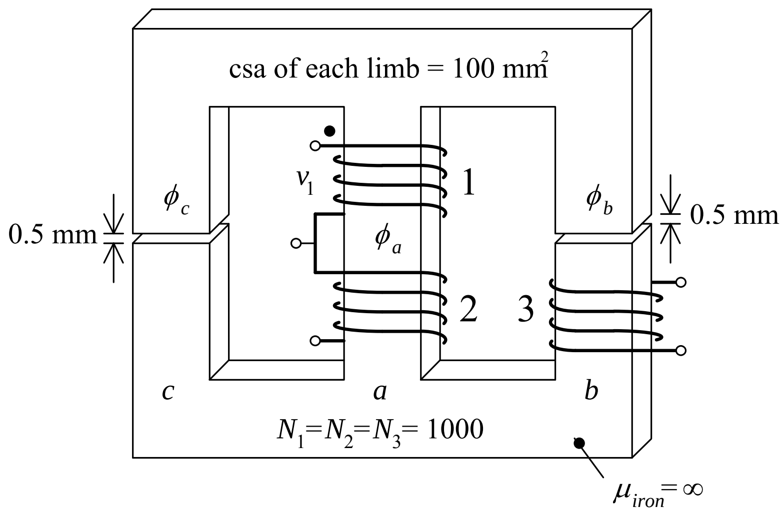
A 115 V RMS, 60 Hz generator supplies 3 kW to a $3\,\Omega$ load via a two winding transformer (assumed to be ideal). Determine the turns ratio and the minimum voltage and current ratings of each winding.

2.



$E_G = 250$ V RMS, $f = 5$ kHz. Determine the transformer turns ratio needed to achieve maximum power into the load.

3.



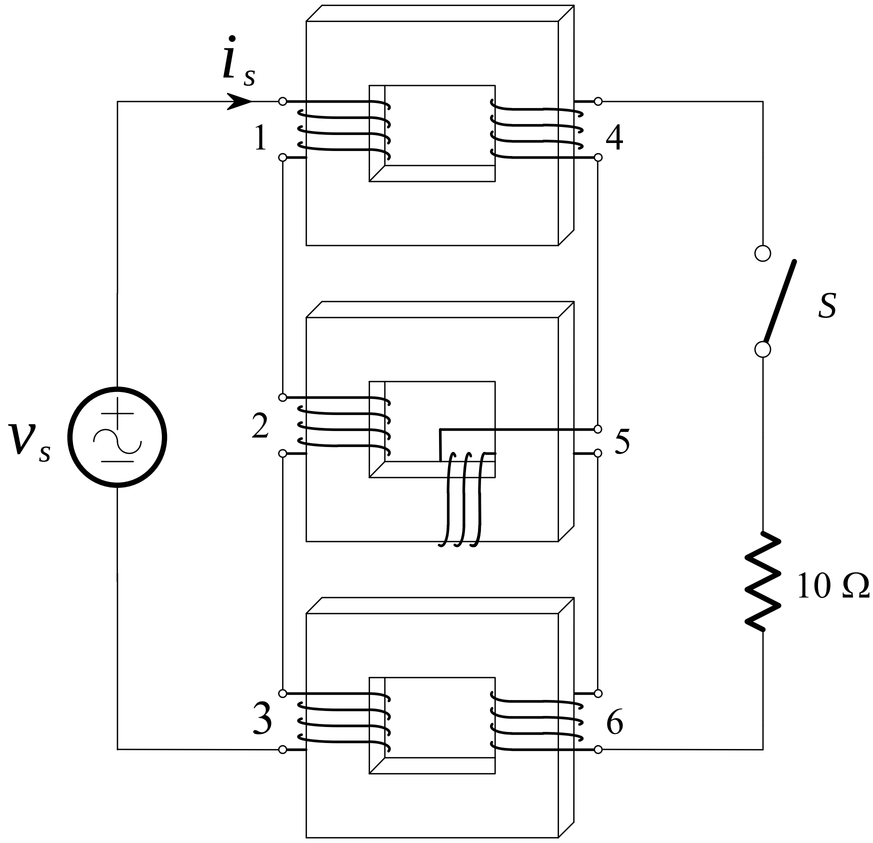
Voltage v_1 is applied to winding 1. A current i_1 results.

(a) Indicate the directions of ϕ_a, ϕ_b and ϕ_c. Place polarity markings (●) on windings 2 and 3.

(b) Draw magnetic and electric equivalent circuits. Calculate the mutual inductances L_{21} and L_{31}.

(c) If $v_1 = \hat{v}_1 \cos(100\pi t)$, determine $\hat{v}_1$ and the peak magnetising current $\hat{i}_1$ needed to give $\hat{B}_a = 1\,\mathrm{T}$.

4.



The three transformers are assumed ideal.

(a) Place appropriate polarity markings (●) on all windings. How many turns has winding 5?

(b) If $v_s = 30\sin(\omega t)$, determine the current in the $10\,\Omega$ resistor and $\hat{i}_s$ when switch S is closed.

(c) What is the value of $\hat{v}_k$, $k = 1\ldots6$?

<u>Hint</u>: Use mmf and voltage / turn balance.