6 Thévenin's Theorem and Norton's Theorem

Contents

6.2

Introduction

Through the use of Thévenin’s theorem and Norton’s theorem, we will see that we can replace a large portion of a complex circuit (often a complicated and uninteresting part) with a very simple equivalent circuit, thus enabling analysis and focus on one particular element of the circuit.

6.1 Thévenin's and Norton's Theorem

Thévenin's and Norton's theorems greatly simplify the analysis of many linear circuits. Léon Thévenin was a French engineer working in telegraphy who first published a statement of the theorem in 1883. Edward Norton was a scientist with the Bell Telephone Laboratories who mentioned his theorem in a technical memorandum in 1926, but never published it.

Thévenin's theorem tells us that it is possible to replace a large portion of a linear circuit (often a complicated and / or uninteresting portion) by an equivalent circuit containing only an independent voltage source in series with a resistor:

Thévenin's theorem allows us to replace part of a circuit with a practical voltage source

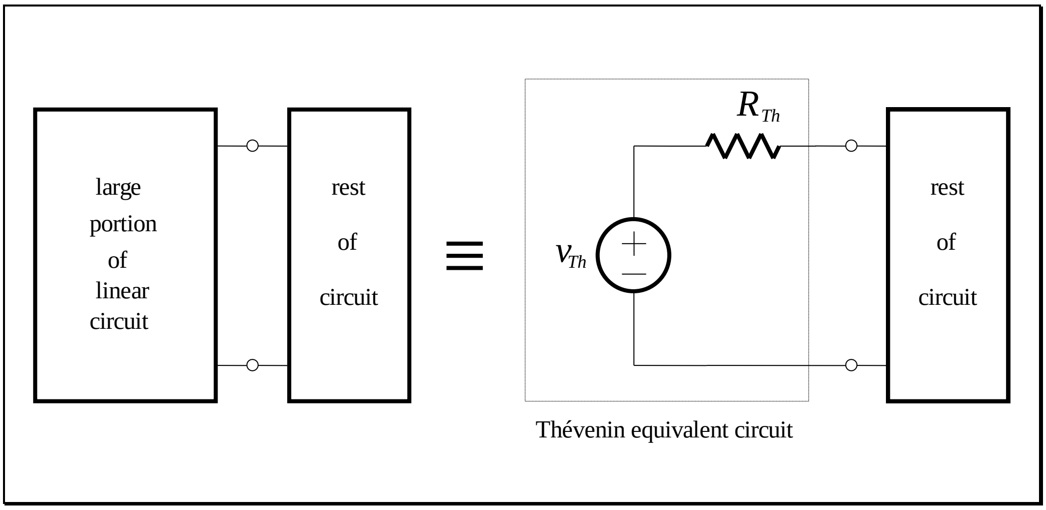


Figure 6.1

Norton's theorem is the dual of Thévenin's theorem, and uses a current source:

Norton's theorem allows us to replace part of a circuit with a practical current source

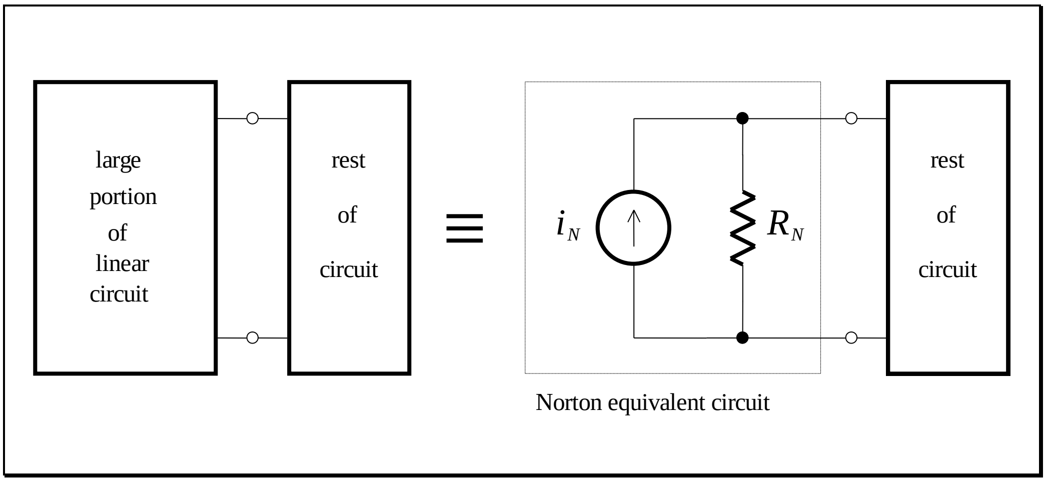


Figure 6.2

6.4

We can state Thévenin's theorem formally as:

Thévenin's theorem

> Given any linear circuit, rearrange it in the form of two circuits *A* and *B* that are connected together at two terminals.
> If either circuit contains a dependent source, its control variable must be in the same circuit.
> Define a voltage v_{oc} as the open-circuit voltage which would appear across the terminals of *A* if *B* were disconnected so that no current is drawn from *A*. Then all the currents and voltages in *B* will remain unchanged if *A* has all its independent sources set to zero and an independent *voltage source* v_{oc} is connected in *series* with the inactive *A* network.

(6.1)

The inactive circuit *A* will always reduce to a single resistor, which we call the Thévenin resistance, R_{Th}. Also, since v_{oc} appears as an independent voltage source in the Thévenin equivalent circuit, it is also denoted as v_{Th}.

We can state Norton's theorem formally as:

Norton's theorem

> Given any linear circuit, rearrange it in the form of two circuits *A* and *B* that are connected together at two terminals.
> If either circuit contains a dependent source, its control variable must be in the same circuit.
> Define a current i_{sc} as the short-circuit current which would appear across the terminals of *A* if *B* were short-circuited so that no voltage is provided by *A*. Then all the voltages and currents in *B* will remain unchanged if *A* has all its independent sources set to zero and an independent *current source* i_{sc} is connected in *parallel* with the inactive *A* network.

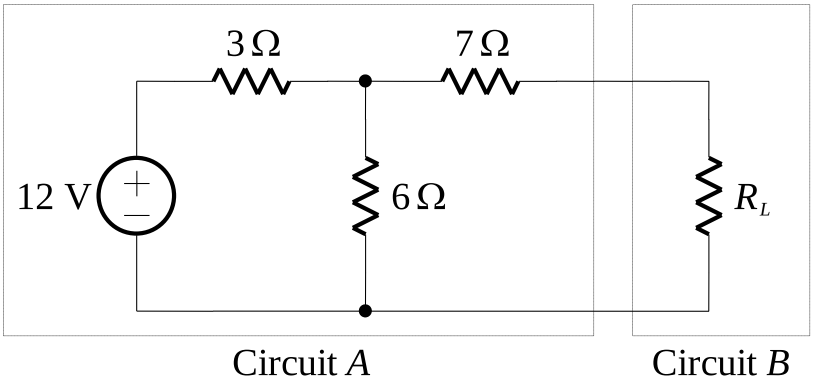
(6.2)

The inactive circuit *A* will always reduce to a single resistor, which we call the Norton resistance, R_N. Also, since i_{sc} appears as an independent current source in the Norton equivalent circuit, it is also denoted as i_N.

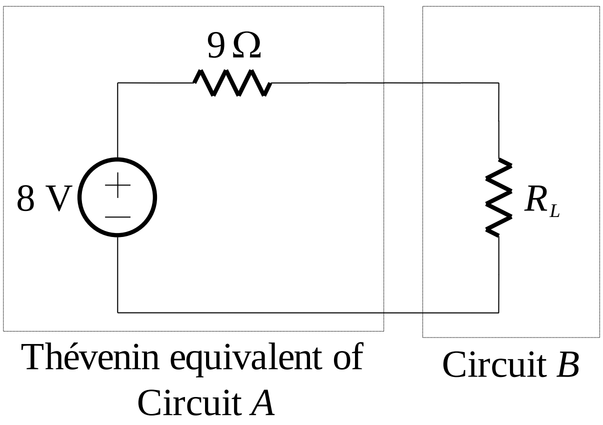
The Thévenin or Norton equivalent circuit allows us to draw a new simpler circuit and make rapid calculations of the voltage, current, and power. It also allows us to easily choose a “load” resistance for maximum power transfer.

EXAMPLE 6.1 Thévenin and Norton Equivalent Circuits

Consider the circuit shown below:



The broken lines separate the original circuit into circuits *A* and *B*. We shall assume that our main interest is in circuit *B*, which consists only of a “load” resistor R_L. To form the Thévenin equivalent circuit, we disconnect circuit *B* and use voltage division to determine that $v_{oc} = 8\text{ V}$. When we set all independent sources in circuit *A* to zero, we replace the 12 V source with a short-circuit. “Looking back” into the inactive *A* circuit, we “see” a $7\,\Omega$ resistor connected in series with the parallel combination of $6\,\Omega$ and $3\,\Omega$. Thus, the inactive *A* circuit can be represented by a $9\,\Omega$ resistor. If we now replace circuit *A* by its Thévenin equivalent circuit, we have:



Note that the Thévenin equivalent circuit we have obtained for circuit *A* is completely independent of circuit *B* – an equivalent for *A* may be obtained *no matter what arrangement of elements is connected to the* A *circuit, even if circuit* B *is nonlinear!*

From the viewpoint of the load resistor R_L, the Thévenin equivalent circuit is identical to the original; from our viewpoint, the circuit is much simpler and we can now easily compute various quantities. For example, the power delivered to the load is:

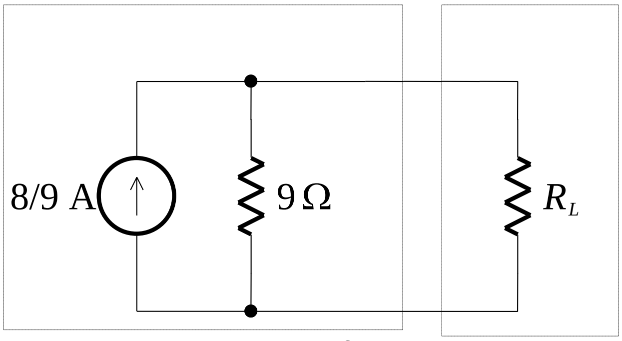
$$P_L = \left(\frac{8}{9 + R_L} \right)^2 R_L$$

Furthermore, we can now easily see that the maximum voltage which can be obtained across R_L is 8 V when $R_L = \infty$. A quick transformation of the Thévenin equivalent circuit to a practical current source (the Norton equivalent) indicates that the maximum current which may be delivered to the load is $8/9\ \text{A}$ for $R_L = 0$. The maximum power transfer theorem shows that a maximum power is delivered to R_L when $R_L = 9\,\Omega$. None of these facts is readily apparent from the original circuit.

To form the Norton equivalent circuit, we short-circuit the *B* circuit and use the current divider rule to discover:

$$i_{sc} = \frac{6}{6+7}\left(\frac{12}{3+\dfrac{6\cdot 7}{6+7}}\right) = \frac{72}{39+42} = \frac{72}{81} = \frac{8}{9}\ \text{A}$$

When we set all independent sources in circuit *A* to zero, we get the same results as for the Thévenin circuit, and so $R_N = 9\,\Omega$. The Norton equivalent circuit is therefore:



Norton equivalent of Circuit *A*

Circuit *B*

It should be apparent from the previous example that we can easily find the Norton equivalent circuit from the Thévenin equivalent circuit, and vice versa, by a simple source transformation. Using our previous results, we must have:

$$\boxed{R_{Th} = R_N} \tag{6.3}$$

The Thévenin and Norton equivalent resistances are the same

Because of this result, we usually just refer to the resistor in either equivalent circuit as the Thévenin resistance, R_{Th}.

We also have:

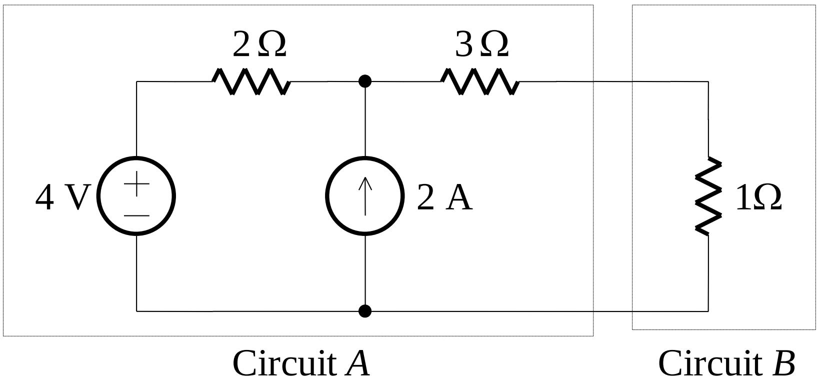
$$\boxed{v_{oc} = R_{Th} i_{sc}} \tag{6.4}$$

The relationship between the Thévenin and Norton equivalent circuits

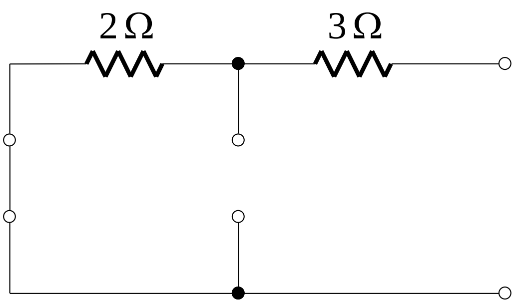
6.8

EXAMPLE 6.2 Thévenin and Norton Equivalent Circuits

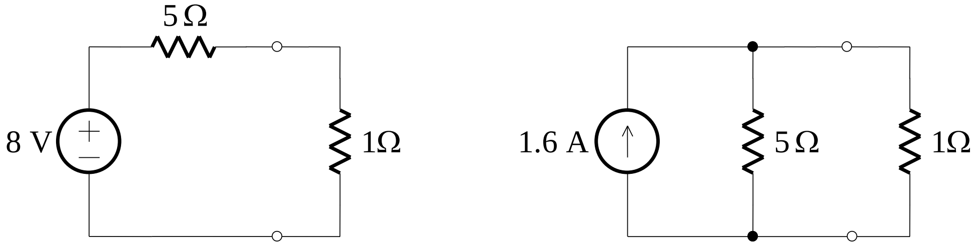
Consider the circuit shown below:



The Thévenin and Norton equivalent circuits are desired from the perspective of the 1Ω resistor. We determine R_{Th} for the inactive network, and then find either v_{oc} or i_{sc}. Making the independent sources inactive, we have:



We thus have $R_{Th} = 5\,\Omega$. The open-circuit voltage can be determined using superposition. With only the 4 V source operating , the open-circuit voltage is 4 V. When only the 2 A source is on, the open-circuit voltage is also 4 V. Thus, with both sources operating, we have $v_{oc} = 8\,\text{V}$. This determines the Thévenin equivalent, and from it the Norton equivalent, shown below:



We will often find it convenient to determine either the Thévenin or Norton equivalent by finding both the open-circuit voltage and the short-circuit current, and then determining the Thévenin resistance by:

The Thévenin equivalent resistance can be obtained from the open-circuit voltage and short-circuit current

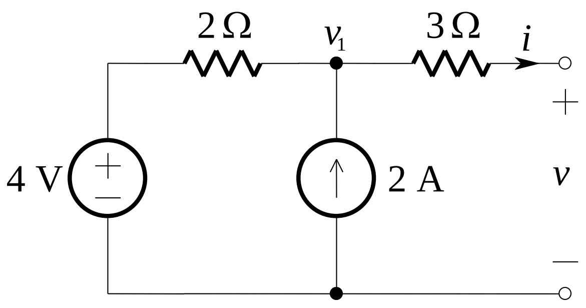
$$R_{Th} = \frac{v_{oc}}{i_{sc}} \tag{6.5}$$

Another method has a certain appeal because it can be used for any circuit. Simply label the terminals of the *A* circuit as *v*, define the current leaving the positive polarity as *i*, then analyse the *A* circuit to obtain an equation in the form $v = v_{oc} - R_{Th} i$.

A method to obtain the Thévenin equivalent circuit that works for *all* circuits

EXAMPLE 6.3 Thévenin Equivalent Using a Linear Equation

Consider the circuit shown below:



KCL at the middle node gives:

$$\frac{v_1 - 4}{2} - 2 + i = 0$$
$$v_1 = 8 - 2i$$

KVL at the output gives:

$$\begin{aligned} v &= v_1 - 3i \\ &= 8 - 5i \\ &= v_{oc} - R_{Th} i \end{aligned}$$

from which $v_{oc} = 8\,\mathrm{V}$ and $R_{Th} = 5\,\Omega$, as before.

6.2 Summary

- Thévenin's theorem tells us that it is possible to replace a large portion of a linear circuit by an equivalent circuit containing only an independent voltage source in series with a resistor.

- Norton's theorem tells us that it is possible to replace a large portion of a linear circuit by an equivalent circuit containing only an independent current source in parallel with a resistor.

- There are several methods that can be applied to determine a Thévenin or Norton equivalent circuit. Some methods are only applicable to certain circuits, and the most convenient analysis method should be chosen.

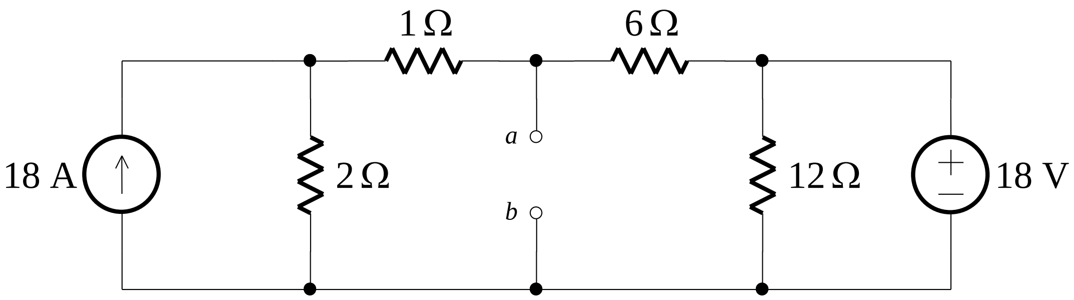
6.3 References

Hayt, W. & Kemmerly, J.: *Engineering Circuit Analysis*, 3rd *Ed*., McGraw-Hill, 1984.

Exercises

1.

Consider the circuit shown below:



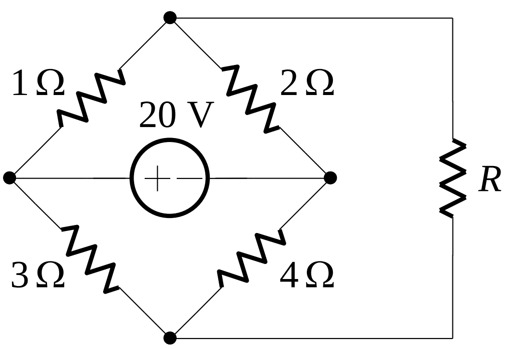
(a) Find the Norton equivalent of the circuit.

(b) If a variable resistor *R* were placed between terminals *a* and *b*, what value would result in maximum power being drawn from the terminals?

(c) Find the maximum power that could be drawn from terminals *a* and *b*.

2.

Find the maximum power that can be delivered to a variable *R* in the circuit below:



Index