Lecture 2 – Power and Symmetrical Components

Per-unit values. Power flow in an interconnector. Modelling of power system loads. Introduction to symmetrical components.

Per-Unit Values of Electrical Quantities

Definition

Per-unit values are used extensively in power system calculations. The per-unit value is defined as the ratio of a physical value of some quantity to a *base* (or reference) value. Thus the relevant physical quantities, like voltage, current, etc. are expressed as pure non-dimensional numbers, traditionally designated by the label "per unit" or "p.u.". Per-unit values are frequently expressed in percent.

$$\boxed{\begin{aligned} \text{p.u. value} &= \frac{\text{physical value}}{\text{base value}} \\ \text{\% value} &= \text{p.u. value} \times 100\% \end{aligned}} \qquad (2.1)$$

p.u. value defined

The "physical value" may be real or complex. The "base value" is always real.

Base Values

To be useful, the p.u. values must obey standard network equations, like Ohm's Law, etc. That means that the base values cannot be chosen independently. The usual practice is to choose the base values for power and voltage, and then calculate compatible base values for current and impedance.

The base power S_{base} (used for *S*, *P*, and *Q*) is usually one of the following:

- Equipment VA rating (either total or "per phase"), when dealing with a single item, or a number of items with equal VA ratings.
- An arbitrarily assigned figure, when dealing with a collection of items with different individual VA ratings. A commonly used figure for large power systems is $S_{base} = 100\,\text{MVA}$ (total three-phase value).

The base voltage V_{base} may be either of the following:

- The nominal voltage of the power system (line-to-line voltage).
- The nominal phase voltage of an equivalent star system, i.e. the nominal system voltage divided by $\sqrt{3}$.

Care is required by the inexperienced to decide which of the two possible base voltages to use in a particular case.

Having decided on S_{base} and V_{base}, the values of base current and base impedance are calculated as in the following table. The table is organised in two columns. Experienced power systems engineers generally use, for three-phase calculations, the formulae in the second column, but students may find it less confusing to stick with the first column, and work with all quantities "per phase".

Table of base values

Single-phase and 3-phase, using phase voltage, phase current and power per phase as base values	3-phase, using line voltage, line current, 3-phase power and equivalent star impedance	
$I_{base} = \dfrac{S_{base}}{V_{base}}$		(2.2)
	$I_{base} = \dfrac{S_{base}}{\sqrt{3}V_{base}}$	(2.3)
$Z_{base} = \dfrac{V_{base}}{I_{base}}$		(2.4)
	$Z_{base} = \dfrac{V_{base}}{\sqrt{3}I_{base}}$	(2.5)
or $Z_{base} = \dfrac{(V_{base})^2}{S_{base}}$	or $Z_{base} = \dfrac{(V_{base})^2}{S_{base}}$	(2.6)

Note that Eq. (2.6) is identical for both columns.

In network calculations all per-unit parameters must be determined on a common value of S_{base} for the entire network. When the network includes transformers, the value of V_{base} are changed for different parts of the network according to the turns ratio of the transformers. Hence I_{base} and Z_{base} are also changed.

A common power base must be used for an entire network

Base Conversion of Impedance

Frequently the p.u. value of impedance $(Z_{p.u.})_{given}$ is known, based on $(S_{p.u.})_{given}$, say the equipment rating, but we need $Z_{p.u.}$ on a different VA base $(S_{base})_{new}$. It is easy to show that:

$$\boxed{(Z_{p.u.})_{new} = (Z_{p.u.})_{given}\left(\frac{(S_{base})_{new}}{(S_{base})_{given}}\right)} \tag{2.7}$$

Advantages of Using Per-Unit Values

Numerical values of p.u. values tend to fall within narrow ranges, making it easier to detect errors. More importantly, the fact that the numerical range of variables is close to “1” means that the numerical methods used in the computer solution of power system equations are able to operate on “well-conditioned” matrices. This is important for large systems where there may be significant errors due to “machine number” round-off. Also, the need to handle very large or small numbers is eliminated. This advantage is lost if we make the value of S_{base} too far removed from equipment ratings.

Calculations involving transformers are simplified, since the p.u. value of impedance is the same whether referred to the high voltage or the low voltage winding.

2.4

Power Flow in an Interconnector

The General Problem

A large power supply network is a collection of many buses (nodes), some connected to generators, and some to loads, but all interconnected via transmission lines and/or transformers. There is a need to simultaneously control node voltages and power flow. In what follows, we focus on a single interconnector between node 1 and node 2 in such a network. We will assume coupling to other interconnectors to be negligible.

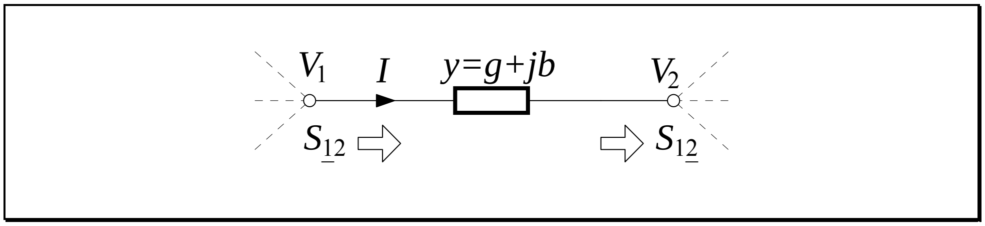


Figure 2.1

Let:

$$\begin{aligned}
V_1 &= \text{voltage (phase voltage) at node 1} \\
V_2 &= \text{voltage (phase voltage) at node 2} \\
\theta_{12} &= \angle V_1 - \angle V_2 \\
y = g + jb &= \text{branch admittance between nodes 1 and 2} \\
S_{\underline{1}2} = P_{\underline{1}2} + jQ_{\underline{1}2} &= \text{power leaving node 1 towards node 2} \\
S_{1\underline{2}} = P_{1\underline{2}} + jQ_{1\underline{2}} &= \text{power from node 1 arriving at node 2}
\end{aligned} \tag{2.8}$$

At node 1:

$$\begin{aligned}
S_{\underline{1}2} &= V_1 I^* = V_1 y^* (V_1 - V_2)^* = y^* \left(|V_1|^2 - V_1 V_2^* \right) \\
&= y^* \left\{ |V_1|^2 - |V_1||V_2| (\cos\theta_{12} + j\sin\theta_{12}) \right\} \\
&= (g - jb) \left\{ \left(|V_1|^2 - |V_1||V_2|\cos\theta_{12} \right) - j\left(|V_1||V_2|\sin\theta_{12} \right) \right\}
\end{aligned} \tag{2.9}$$

$$P_{\underline{1}2} = g\left(|V_1|^2 - |V_1||V_2|\cos\theta_{12} \right) - b|V_1||V_2|\sin\theta_{12} \quad \text{W/phase} \tag{2.10}$$

$$Q_{\underline{1}2} = -g|V_1||V_2|\sin\theta_{12} + b\left(|V_1||V_2|\cos\theta_{12} - |V_1|^2 \right) \quad \text{var/phase} \tag{2.11}$$

At node 2:

$$\begin{aligned} S_{1\underline{2}} &= V_2 I^* = V_2 y^* \left(V_1 - V_2\right)^* = y^* \left(V_1^* V_2 - \left|V_2\right|^2\right) \\ &= y^* \left\{ \left|V_1\right| \left|V_2\right| \left(\cos\theta_{12} - j\sin\theta_{12}\right) - \left|V_2\right|^2 \right\} \\ &= \left(g - jb\right) \left\{ \left(\left|V_1\right| \left|V_2\right| \cos\theta_{12} - \left|V_2\right|^2 \right) - j\left(\left|V_1\right| \left|V_2\right| \sin\theta_{12} \right) \right\} \end{aligned} \tag{2.12}$$

$$P_{1\underline{2}} = g\left(\left|V_1\right| \left|V_2\right| \cos\theta_{12} - \left|V_2\right|^2 \right) - b\left|V_1\right| \left|V_2\right| \sin\theta_{12} \quad \text{W/phase} \tag{2.13}$$

$$Q_{1\underline{2}} = -g\left|V_1\right| \left|V_2\right| \sin\theta_{12} - b\left(\left|V_1\right| \left|V_2\right| \cos\theta_{12} - \left|V_2\right|^2 \right) \quad \text{var/phase} \tag{2.14}$$

The complex power lost in the interconnector is:

$$S_{1\underline{2}} - S_{\underline{1}2} = y^* \left|V_1 - V_2\right|^2 = \left(g - jb\right) \left|V_1 - V_2\right|^2 \tag{2.15}$$

and by the cosine rule:

$$S_{1\underline{2}} - S_{\underline{1}2} = \left(g - jb\right) \left(\left|V_1\right|^2 + \left|V_2\right|^2 - 2\left|V_1\right| \left|V_2\right| \cos\theta_{12} \right) \tag{2.16}$$

The result in Eq. (2.16) can also be obtained by subtracting Eq. (2.12) from Eq. (2.9).

The Ideal "Loss Free" Interconnector

Let the interconnector be a pure inductor with impedance jX. Then $g = 0$ and $b = X^{-1}$. From Eqs. (2.10), (2.11), (2.13) and (2.14):

Power flow in an ideal "loss free" interconnector

$$\boxed{\begin{aligned} P_{\underline{1}2} = P_{1\underline{2}} &= \frac{|V_1||V_2|}{X}\sin\theta_{12} && \text{W/phase} \\ Q_{\underline{1}2} &= \frac{|V_1|^2 - |V_1||V_2|\cos\theta_{12}}{X} && \text{var/phase} \\ Q_{1\underline{2}} &= \frac{|V_1||V_2|\cos\theta_{12} - |V_2|^2}{X} && \text{var/phase} \end{aligned}} \tag{2.17}$$

In an inductive circuit real power does not necessarily flow from the higher voltage node to the lower voltage node

There is no real power loss, although there is a reactive power loss. The direction of real power flow is determined entirely by θ_{12}. Positive θ_{12} means V_1 leads V_2 and power flows from node 1 to node 2. The angle θ_{12} is known as the *power angle*, in machine theory also as the *torque angle*.

The average transferred reactive power is:

$$Q_{av} = \frac{Q_{\underline{1}2} + Q_{1\underline{2}}}{2} = \frac{|V_1|^2 - |V_2|^2}{2X} \tag{2.18}$$

which indicates that the reactive power flows from the node with the higher voltage towards the node with the lower voltage, and this flow is not dependent on θ_{12}. In the special case when $|V_1| = |V_2|$ there is no average transferred reactive power flow, and the reactive power losses are supplied equally from both ends.

Example

An interconnector has the following known quantities:

$$|V_1| = |V_2| = \frac{138}{\sqrt{3}}\ \mathrm{kV}\ ,\ X = 80\ \Omega/\mathrm{ph}\ .$$

Determine the power angle θ_{12} and the reactive flow in the line ends if the active power flow from 1 to 2 is 100 MW.

Since $|V_1| = |V_2| = |V|$, then $P_{\underline{1}2} = P_{1\underline{2}} = \frac{|V|^2}{X}\sin\theta_{12}$. Therefore:

$$\sin\theta_{12} = \frac{100}{3}\frac{3}{138^2}80 = 0.4208 \qquad \theta_{12} = 24.84°$$

Then the reactive power flows are:

$$Q_{\underline{1}2} = \frac{|V|^2}{X}\left(1-\cos\theta_{12}\right) \quad \mathrm{var/ph}, \quad Q_{\underline{1}2} = -Q_{1\underline{2}}$$

$$Q_{\underline{1}2} = 3\frac{138^2}{3}\frac{1}{80}\left(1-\cos 24.84°\right) = 22.02\ \mathrm{Mvar}$$

$$Q_{1\underline{2}} = -22.02\ \mathrm{Mvar}$$

Note the cancellation of threes and powers of ten in the example.

The Static Stability Limit

Differentiating Eq. (2.13) with respect to θ_{12}:

$$\frac{\partial P_{1\underline{2}}}{\partial \theta_{12}} = -g|V_1||V_2|\sin\theta_{12} - b|V_1||V_2|\cos\theta_{12} = 0 \tag{2.19}$$

for maximum $P_{1\underline{2}}$. Therefore the maximum $P_{1\underline{2}}$ occurs when:

$$\tan\theta_{12} = -\frac{b}{g} = \frac{X}{R} \tag{2.20}$$

and this power is known as the *static stability limit*. For a purely inductive circuit, $R = 0$, and the static stability limit occurs when $\theta_{12} = \pi/2$. Hence for the purely inductive circuit:

$$\left(P_{1\underline{2}}\right)_{\max} = \frac{|V_1||V_2|}{X} \quad \text{W} \tag{2.21}$$

In a practical power system the voltages are confined to fairly narrow limits, so that $|V_1| \approx |V_2| \approx |V|$, and we have:

Static stability limit for an ideal "loss free" interconnector

$$\boxed{\left(P_{1\underline{2}}\right)_{\max} \approx \frac{|V|^2}{X} \quad \text{W}} \tag{2.22}$$

Eq. (2.22) shows why high voltages are required for transmitting large amounts of power over long distances (large *X*).

Role of vars in the Power System

Clearly, the role of the power system is to supply energy, and hence real power (watts). The inductive effect of transmission lines, transformers, and most loads, dictates that these consume vars, and this inevitable reactive power must also be supplied to satisfy the law of conservation of complex power.

The previous example illustrates that the vars also have a role of promoting the transmission of real power while keeping the supply voltage within the required tolerances. In the example it is not enough to supply vars to the transmission line from the sending end alone – we also need a separate source of vars at the receiving end.

The vars can be supplied by synchronous machines (generators or motors), or by capacitors. A capacitor, or any load with a leading power factor, consumes negative vars, and this is equivalent to generating positive vars.

Modelling of Power System Loads

General

Power system loads may be of many different types, such as motors, heaters, lighting and electronic equipment. Most system loads are a mixture of different types.

The loads vary in size (watts and vars), symmetry, daily and seasonal variations, short term fluctuations (e.g. arc furnaces, woodchip mills). Some nonlinear loads (e.g. rectifiers and other power conversion equipment) may also produce significant harmonic currents.

Most loads vary with changing voltage and frequency.

Variation with Voltage

Assume:

$$P = k|\mathbf{V}|^n \tag{2.23}$$

Consider a small change $\delta|\mathbf{V}|$ in the voltage magnitude. The *relative* change is $\delta|\mathbf{V}|/|\mathbf{V}|$. A first-order Taylor Series approximation gives the change in power as:

$$\delta P \approx \frac{\partial P}{\partial|\mathbf{V}|}\delta|\mathbf{V}| = kn|\mathbf{V}|^{n-1}\delta|\mathbf{V}| = n\frac{P}{|\mathbf{V}|}\delta|\mathbf{V}| \tag{2.24}$$

Hence the *relative* change in power is:

Relative change in power for a voltage variation

$$\boxed{\frac{\delta P}{P} \approx n\frac{\delta|\mathbf{V}|}{|\mathbf{V}|}} \tag{2.25}$$

While the above is derived for the power *P*, clearly the same relation applies to the reactive power *Q*, possibly with a different value of the exponent *n*.

Variation with Frequency

Let the power P be some function F of frequency f, $P = F(f)$. Then for a small change in frequency δf we have, using a first-order Taylor Series approximation:

$$\delta P \approx \frac{\partial P}{\partial f} \delta f \tag{2.26}$$

and therefore the relative change in power is:

$$\boxed{\frac{\delta P}{P} \approx \left(\frac{f}{P} \frac{\partial P}{\partial f} \right) \frac{\partial f}{f}} \tag{2.27}$$

Relative change in power for a frequency variation

Examples

(1) Constant impedance load

Admittance $Y = 1/Z = g + jb = \text{constant}$ (b is negative for inductive load).

$S = Y^* |\mathbf{V}|^2$, therefore $P = g|\mathbf{V}|^2$ and $Q = -b|\mathbf{V}|^2$.

Therefore, $n = 2$ in Eq. (2.25). Thus a 1% drop in voltage results in a 2% drop in both P and Q.

(2) Incandescent lighting load

The resistance of light globes increases significantly with increasing operating temperature, therefore the exponent $n < 2$ in Eqs. (2.23) and (2.25). A value of $n = 1.6$ is typical. Hence a 1% drop in voltage results in a 1.6% drop in P (drop in Q is negligible).

(3) Fluorescent lighting load

The $P \sim |\mathbf{V}|$ relationship is more complicated than it is for incandescent lights, but in absence of better information, $n = 1.6$ may be assumed.

(4) Synchronous motor load

The speed of the motor is not affected by small changes of voltage, but is proportional to frequency. However, when the voltage drops too low the motor loses synchronism.

As the mechanical load is unaffected by voltage, the electrical power *P* may also be assumed to remain constant. Therefore $n \approx 0$ in Eqs. (2.23) and (2.25).

Variation of *P* with frequency depends on how the mechanical load varies with speed.

(5) Induction motor load

Induction motor torque-slip characteristics give torque $T \propto s|\mathbf{V}|^2$, where *s* = slip, but the torque is actually a characteristic of the mechanical load. If for example, *T* is constant, then $s \propto |\mathbf{V}|^{-2}$. In practice $s < 0.05$ at rated conditions.

Say $s = 0.05$, and we increase the voltage by 1%. The new slip is then $0.05(1.01)^{-2} = 0.049$. Speed $\propto (1-s)$, therefore with constant *T* the mechanical power is also proportional to $(1-s)$. Hence the mechanical power increases by the ratio $\dfrac{1-0.049}{1-0.05} = 1.00105$, i.e. the mechanical power increases by only 0.105% for a 1% voltage increase.

For an ideal induction motor $(1-s)$ is equal to the theoretical efficiency, hence the improved efficiency at the lower slip exactly compensates for the increased mechanical power, so that the electrical power *P* remains constant in this ideal case. This is not quite so for a practical motor, particularly as the torque varies with speed, but we would generally be justified in assuming that the real power *P* is independent of voltage for an induction motor. So $n \approx 0$ in Eqs. (2.23) and (2.25) for *P*. This is not true for the reactive power *Q*, which increases with voltage $(n > 0)$.

Symmetrical Components – Introduction

Impedance and Admittance Matrices of a Three-Phase Network

Let the “network” be a three-phase four-terminal circuit as shown:

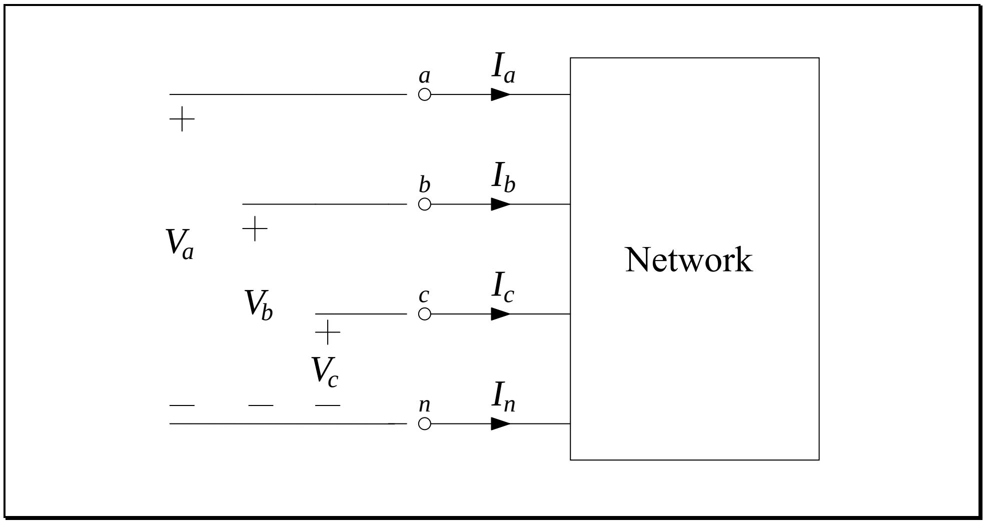


Figure 2.2

Assume that the network contains no internal sources. Whatever the structure of the network, and regardless of how many branches and nodes it contains, its external behaviour is determined by KCL and a 3 x 3 matrix, either $\mathbf{Z}_{abc}$ or $\mathbf{Y}_{abc}$. KCL gives:

$$I_a + I_b + I_c + I_n = 0 \tag{2.28}$$

We can regard the network as having three loops (*a*, *b*, *c*) completed via the common “*n*” terminal. The loop currents and applied voltages are then related by the impedance matrix $\mathbf{Z}_{abc}$:

$$\mathbf{V}_{abc} = \mathbf{Z}_{abc}\mathbf{I}_{abc} \tag{2.29}$$

Alternatively, we can regard the network as having three independent nodes (*a*, *b*, *c*) with the "*n*" terminal as the reference node. The node voltages and applied node currents are then related by the admittance matrix $\mathbf{Y}_{abc}$:

$$\mathbf{I}_{abc} = \mathbf{Y}_{abc}\mathbf{V}_{abc} \tag{2.30}$$

Eqs. (2.29) and (2.30) describe the same network, therefore $\mathbf{Z}_{abc} = \mathbf{Y}_{abc}^{-1}$ and $\mathbf{Y}_{abc} = \mathbf{Z}_{abc}^{-1}$, providing the inverse matrices exist in each case. In the general case:

$$\mathbf{Z}_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \tag{2.31}$$

$\mathbf{Y}_{abc}$ is the same as Eq. (2.31) with the *Z*'s replaced by *Y*'s.

In the general case the above equations can be quite difficult to solve.

<u>For a symmetrical three-phase power apparatus:</u>

$$\begin{aligned} Z_{aa} &= Z_{bb} = Z_{cc} = a \\ Z_{ab} &= Z_{bc} = Z_{ca} = b \\ Z_{ba} &= Z_{cb} = Z_{ac} = c \end{aligned} \tag{2.32}$$

Thus, in the symmetrical case, the matrix Eq. (2.31) takes the specialised form as follows:

$$\mathbf{Z}_{abc} = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix} \tag{2.33}$$

and the form of $\mathbf{Y}_{abc}$ is also similar.

Diagonalization of the Impedance and Admittance Matrices

It is useful to change the reference frame of the impedance matrix Eq. (2.33) so as to eliminate all off-diagonal elements. This involves solving the characteristic equation:

$$\begin{vmatrix} (a-Z) & b & c \\ c & (a-Z) & b \\ b & c & (a-Z) \end{vmatrix} = 0 \tag{2.34}$$

hence:

$$(a-Z)^3 - 3bc(a-Z) + b^3 + c^3 = 0 \tag{2.35}$$

We now introduce subscripts (0, 1, 2) for the three solutions of the cubic equation. The solutions are:

$$\begin{aligned} Z_0 &= a + b + c \\ Z_1 &= a + h^2 b + hc \\ Z_2 &= a + hb + h^2 c \end{aligned} \tag{2.36}$$

where a, b, c are as defined in Eq. (2.32).

The next step is to determine the "eigenvectors" by solving:

$$\mathbf{Z}_{abc}\mathbf{H}_i = Z_i \mathbf{U}_3 \mathbf{H}_i \tag{2.37}$$

where:

$$\begin{aligned} &i = (0,1,2) \\ &\mathbf{U}_3 = 3 \text{ x } 3 \text{ identity matrix} \\ &\mathbf{H}_i = \text{eigenvector (3 x 1 matrix) corresponding to } Z_i \end{aligned} \tag{2.38}$$

A set of three solutions is:

$$\mathbf{H}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{H}_1 = \begin{bmatrix} 1 \\ h^2 \\ h \end{bmatrix}, \quad \mathbf{H}_2 = \begin{bmatrix} 1 \\ h \\ h^2 \end{bmatrix} \tag{2.39}$$

These solutions are not unique, as any solution multiplied by a complex constant is also a solution.

The eigenvectors of Eq. (2.39) each represent a system of three symmetrical unit length phasors (a, b, c from top down):

$$\boxed{\begin{aligned} &\mathbf{H}_0 = \text{"zero sequence": a, b, c are in phase} \\ &\mathbf{H}_1 = \text{"positive sequence": phase sequence is a, b, c} \\ &\mathbf{H}_2 = \text{"negative sequence": phase sequence is a, c, b} \end{aligned}} \tag{2.40}$$

The three eigenvectors given by (2.39) are the standard basis vectors of the symmetrical components. The corresponding transformation matrix is:

Transformation matrix for converting sequence coordinates to phase coordinates

$$\boxed{\mathbf{H} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_1 & \mathbf{H}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & h^2 & h \\ 1 & h & h^2 \end{bmatrix}} \tag{2.41}$$

Inverting Eq. (2.41) we obtain the inverse transformation matrix:

Transformation matrix for converting phase coordinates to sequence coordinates

$$\boxed{\mathbf{H}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & h & h^2 \\ 1 & h^2 & h \end{bmatrix}} \tag{2.42}$$

(Remember, $h^2 = h^*$)

The factor $1/3$ appears because the eigenvectors are not normalised. In this case the Euclidean norm of $\mathbf{H}_i$ is $\sqrt{3}$, whereas if the eigenvectors were normalised the Euclidean norm would be 1, and the factor $1/3$ would not appear in Eq. (2.42).

The voltages and currents in the original *phase* (a, b, c) reference frame, and the new *sequence* reference frame (0, 1, 2) are related by the following:

$$\boxed{\begin{aligned}\mathbf{V}_{abc} &= \mathbf{H}\mathbf{V}_{012}\\ \mathbf{I}_{abc} &= \mathbf{H}\mathbf{I}_{012}\end{aligned}} \tag{2.43}$$

Transforms from sequence coordinates to phase coordinates

Inverting Eq. (2.43):

$$\boxed{\begin{aligned}\mathbf{V}_{012} &= \mathbf{H}^{-1}\mathbf{V}_{abc}\\ \mathbf{I}_{012} &= \mathbf{H}^{-1}\mathbf{I}_{abc}\end{aligned}} \tag{2.44}$$

Transforms from phase coordinates to sequence coordinates

Combining Eqs. (2.29), (2.43) and (2.44) we obtain:

$$\mathbf{V}_{012} = \mathbf{H}^{-1}\mathbf{Z}_{abc}\mathbf{H}\mathbf{I}_{012} \tag{2.45}$$

or:

$$\boxed{\mathbf{V}_{012} = \mathbf{Z}_{012}\mathbf{I}_{012}} \tag{2.46}$$

where:

$$\boxed{\mathbf{Z}_{012} = \mathbf{H}^{-1}\mathbf{Z}_{abc}\mathbf{H}} \tag{2.47}$$

Transform from phase impedances to sequence impedances

Eq. (2.47) is quite general and applies to symmetrical as well as unsymmetrical impedances. If, however, $\mathbf{Z}_{abc}$ is symmetrical, and therefore conforms to Eq. (2.33), then combining Eq. (2.33) and Eq. (2.47) we obtain for a symmetrical system:

$$\mathbf{Z}_{012} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \tag{2.48}$$

where Z_0, Z_1 and Z_2 are the *sequence impedances* as given by Eq. (2.36).

For a symmetrical system we obtain from Eqs. (2.46) and (2.48):

$$\mathbf{V}_{012} = \mathbf{Z}_{012}\mathbf{I}_{012} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_0 I_0 \\ Z_1 I_1 \\ Z_2 I_2 \end{bmatrix} \tag{2.49}$$

Hence $V_0 = Z_0 I_0$, $V_1 = Z_1 I_1$ and $V_2 = Z_2 I_2$ for a symmetrical network that does not contain sources.

The theory of symmetrical components was developed by diagonalizing the impedance matrix of a symmetrical network. The same can be done with the admittance matrix, giving the sequence admittance matrix:

$$\mathbf{Y}_{012} = \mathbf{H}^{-1}\mathbf{Y}_{abc}\mathbf{H} = \mathbf{Z}_{012}^{-1} \tag{2.50}$$

Passive Circuits

The mutual impedances (or admittances) b and c in Eqs. (2.32) and (2.33) have different values for rotating machines, but for a passive symmetrical network $b = c$ (reciprocity). Then Eq. (2.36) is reduced to:

$$\begin{aligned} Z_0 &= a + 2b \\ Z_1 &= Z_2 = a - b \end{aligned} \tag{2.51}$$

Thus for passive symmetrical circuits the positive and negative sequence impedances are equal, and identical to the effective impedance per phase.

Note on the Effect of the Reference Phase

The eigenvectors in Eq. (2.39), and the transformation matrices Eqs. (2.41) and (2.42) are based on phase "a" providing the zero reference angle. The same could have been done using phase "b" or "c" as a reference. The three sets of results are:

$$\begin{aligned}
\mathbf{H} = \mathbf{H}_a &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & h^2 & h \\ 1 & h & h^2 \end{bmatrix} \quad \mathbf{H}_a^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & h & h^2 \\ 1 & h^2 & h \end{bmatrix} = \frac{1}{3}\mathbf{H}_a^{T*} = \frac{1}{3}\mathbf{H}_a^{*} \\
\mathbf{H}_b &= \begin{bmatrix} 1 & h & h^2 \\ 1 & 1 & 1 \\ 1 & h^2 & h \end{bmatrix} \quad \mathbf{H}_b^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ h^2 & 1 & h \\ h & 1 & h^2 \end{bmatrix} = \frac{1}{3}\mathbf{H}_b^{T*} \\
\mathbf{H}_c &= \begin{bmatrix} 1 & h^2 & h \\ 1 & h & h^2 \\ 1 & 1 & 1 \end{bmatrix} \quad \mathbf{H}_c^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ h & h^2 & 1 \\ h^2 & h & 1 \end{bmatrix} = \frac{1}{3}\mathbf{H}_c^{T*}
\end{aligned} \tag{2.52}$$

These results are included her for completeness, but we will not make any practical use of them. Note that $\mathbf{H}_a^T = \mathbf{H}_a$, but the same is not true in other cases.

Graphical Representation

Define three symmetrical sets of phasor, based on the eigenvectors, as follows:

Zero sequence	Positive sequence	Negative sequence
$\begin{bmatrix} V_{a0} \\ V_{b0} \\ V_{c0} \end{bmatrix} = V_0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \end{bmatrix} = V_1 \begin{bmatrix} 1 \\ h^2 \\ h \end{bmatrix}$	$\begin{bmatrix} V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix} = V_2 \begin{bmatrix} 1 \\ h \\ h^2 \end{bmatrix}$

(2.53)

Then for an arbitrary set of three unsymmetrical phasors V_a, V_b and V_c we expand as follows:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & h^2 & h \\ 1 & h & h^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_0 + V_1 + V_2 \\ V_0 + h^2 V_1 + h V_2 \\ V_0 + h V_1 + h^2 V_2 \end{bmatrix} = \begin{bmatrix} V_{a0} \\ V_{b0} \\ V_{c0} \end{bmatrix} + \begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \end{bmatrix} + \begin{bmatrix} V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix} \tag{2.54}$$

Thus the set of unsymmetrical phasors is expressed as the sum of three sets of symmetrical phasors, or *symmetrical components* as shown below.

Graphical representation of symmetrical components

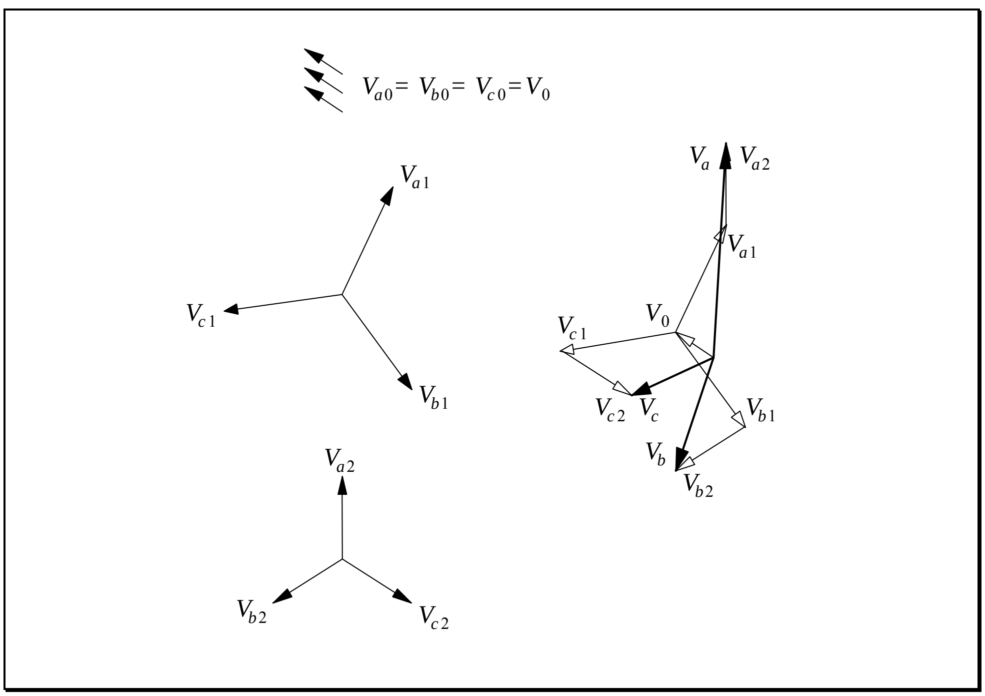


Figure 2.3

The Three-Phase Generator

The figure below illustrates a three-phase generator, assumed to be star-connected.

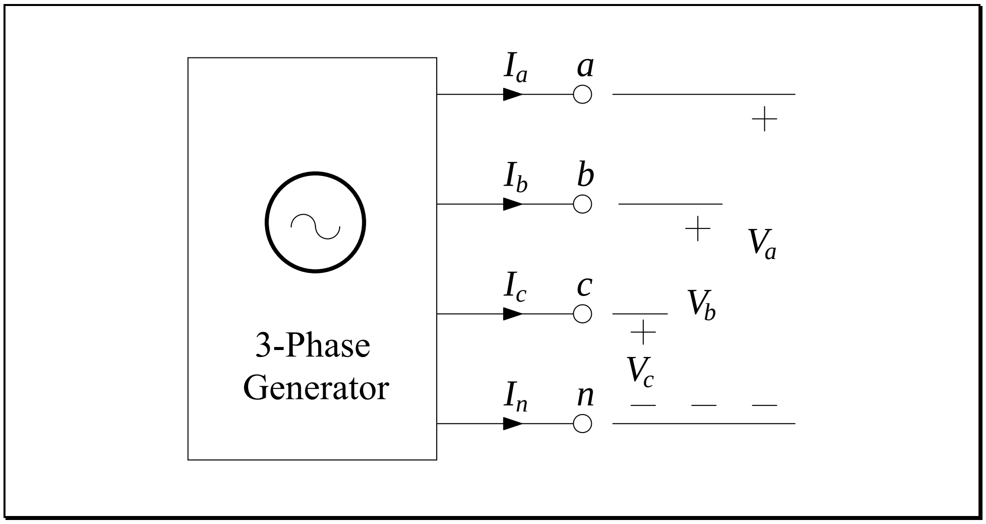


Figure 2.4

The open-circuit phase voltages (emf's) are:

$$\mathbf{E}_{abc} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = E_a \begin{bmatrix} 1 \\ h^2 \\ h \end{bmatrix} \tag{2.55}$$

The terminal voltage, in phase coordinates, is:

$$\mathbf{V}_{abc} = \mathbf{E}_{abc} - \mathbf{Z}_{abc}\mathbf{I}_{abc} \tag{2.56}$$

where $\mathbf{Z}_{abc}$ is the phase impedance matrix of the generator. Pre-multiply both sides with $\mathbf{H}^{-1}$:

$$\mathbf{H}^{-1}\mathbf{V}_{abc} = \mathbf{H}^{-1}\mathbf{E}_{abc} - \mathbf{H}^{-1}\mathbf{Z}_{abc}\mathbf{I}_{abc} \tag{2.57}$$

Using Eqs. (2.43) and (2.44), we get:

$$\mathbf{V}_{012} = \mathbf{E}_{012} - \mathbf{H}^{-1}\mathbf{Z}_{abc}\mathbf{H}\mathbf{I}_{012} \tag{2.58}$$

and then use Eq. (2.47):

$$\mathbf{V}_{012} = \mathbf{E}_{012} - \mathbf{Z}_{012}\mathbf{I}_{012} \tag{2.59}$$

As $\mathbf{Z}_{abc}$ is of the form in Eq. (2.33), $\mathbf{Z}_{012}$ is diagonal. Hence:

$$\mathbf{Z}_{012}\mathbf{I}_{012} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_0 I_0 \\ Z_1 I_1 \\ Z_2 I_2 \end{bmatrix}$$

$$\mathbf{E}_{012} = \mathbf{H}^{-1}\mathbf{E}_{abc} = \frac{E_a}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & h^2 & h \\ 1 & h & h^2 \end{bmatrix} \begin{bmatrix} 1 \\ h^2 \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} \tag{2.60}$$

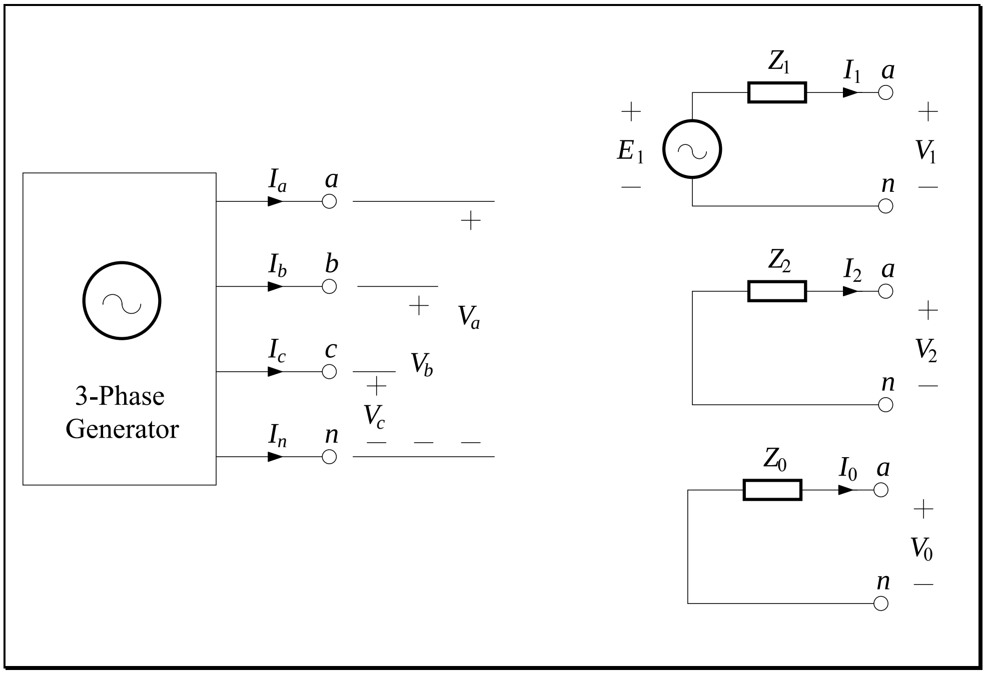
Therefore:

Three-phase star connected generator terminal sequence voltage

$$\boxed{\mathbf{V}_{012} = \mathbf{E}_{012} - \mathbf{Z}_{012}\mathbf{I}_{012} = \begin{bmatrix} -Z_0 I_0 \\ E_1 - Z_1 I_1 \\ -Z_2 I_2 \end{bmatrix}} \tag{2.61}$$

where $E_1 = E_a =$ positive sequence open-circuit voltage.

Eq. (2.61) can be represented by a three-part equivalent circuit as shown below. These are called *sequence networks*.



Three-phase star connected generator equivalent sequence networks

Figure 2.5

The Effect of Neutral Impedance

The neutral terminal may be connected via any value of series impedance ranging from zero to infinity (open circuit). This applies to loads, transformers, generators, etc. We will examine here the case of a generator with its neutral connected to earth via an impedance Z_n, and we define the phase voltages with respect to earth rather than the generator neutral.

Three-phase star connected generator with neutral earthing impedance equivalent sequence networks

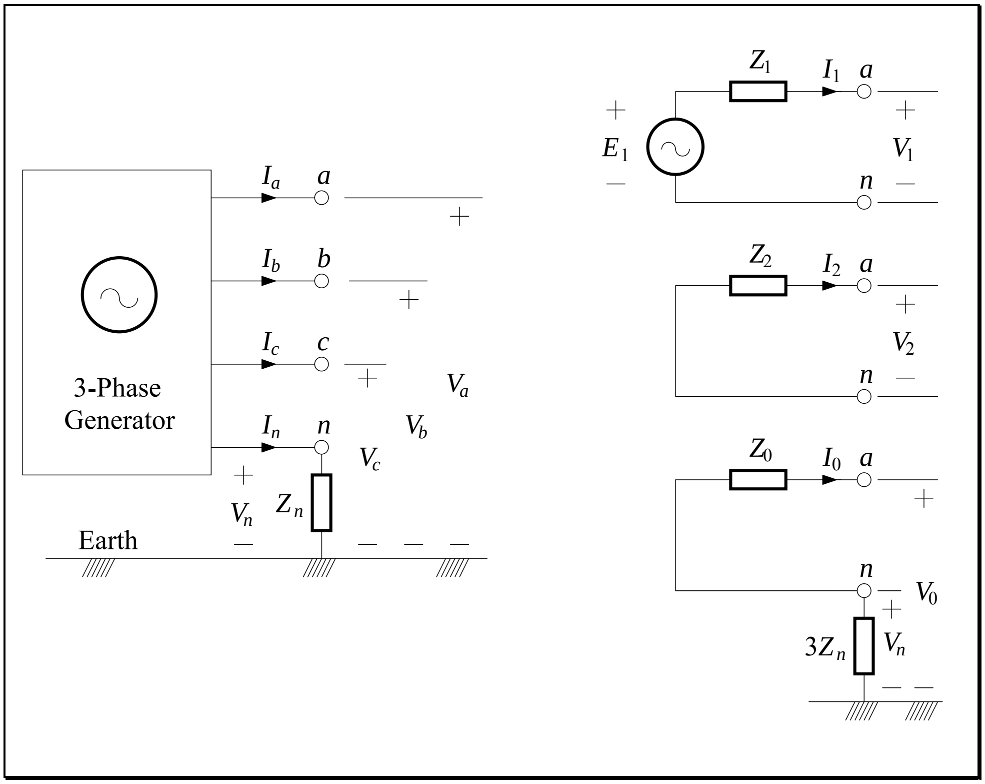


Figure 2.6

The neutral current is:

$$I_n = -(I_a + I_b + I_c) = -3I_0 \tag{2.62}$$

Only the zero sequence current contributes to the neutral current, therefore I_n has no effect on the positive and negative sequence networks.

The total zero sequence voltage is now:

$$\begin{aligned} V_0 &= -Z_0 I_0 + V_n \\ &= -Z_0 I_0 + Z_n I_n \\ &= -Z_0 I_0 - 3Z_n I_0 \end{aligned} \tag{2.63}$$

Therefore:

$$\boxed{V_0 = -(Z_0 + 3Z_n) I_0} \tag{2.64}$$

Here Z_0 is the zero sequence impedance of the generator itself, and $(Z_0 + 3Z_n)$ is the zero sequence impedance of the generator complete with the neutral earthing impedance.

Note that if the neutral is open-circuited, then the zero sequence network is also open-circuited.

Using the Sequence Networks

In a completely symmetrical three-phase power system the positive, negative and zero sequence networks are separate (uncoupled). If now an unsymmetrical condition occurs (accidentally) at just one location, then this condition can be translated into an interconnection between the networks. We will look at some specific cases here. As the symmetrical components are most frequently used for fault calculations, we assume the conditions to be faults, but the results can be applied to similar unbalanced conditions which are not necessarily faults.

Symmetrical Three-Phase Fault

Let the equivalent star fault (or load) impedance be Z_F for the three phases:

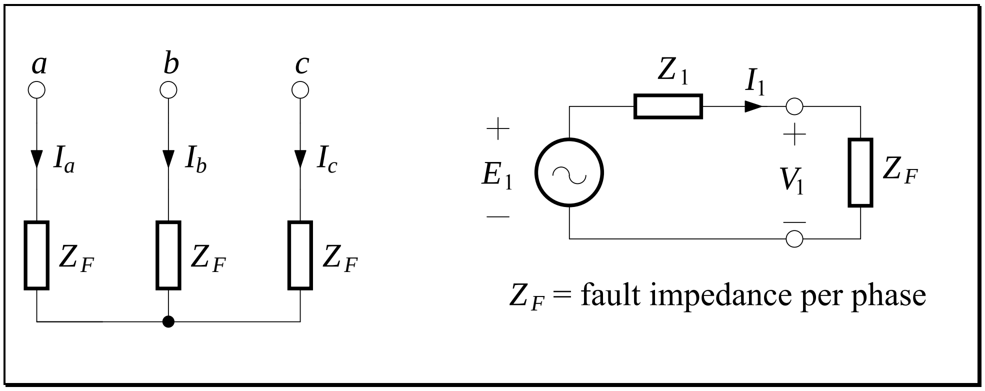


Figure 2.7

Clearly, only positive sequence currents exist in this case. Only the positive sequence network is used, and the analysis is identical to the normal "per phase" analysis of a symmetrical network. The fault current is:

$$\boxed{I_a = I_1 = \frac{E_1}{Z_1 + Z_F}} \tag{2.65}$$

Line-to-Line Fault

Let the fault (or single phase load) be between lines "b" and "c", and have an impedance Z_F :

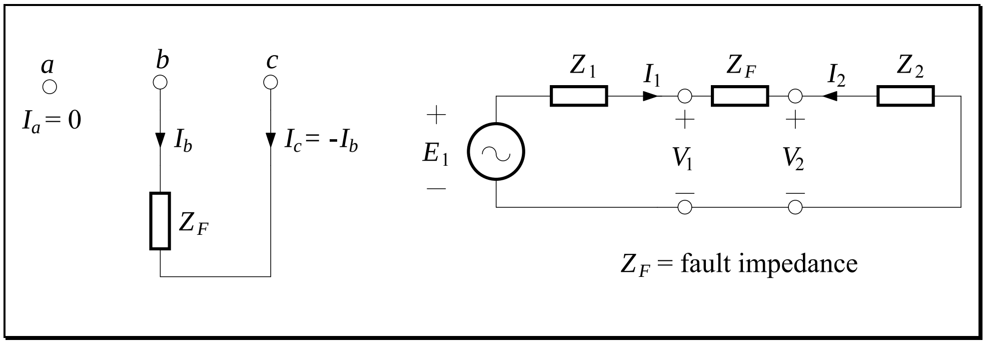


Figure 2.8

The fault admittance matrix is, by inspection:

$$\mathbf{Y}_{abc} = \frac{1}{Z_f}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \tag{2.66}$$

Then:

$$\mathbf{Y}_{012} = \mathbf{H}^{-1}\mathbf{Y}_{abc}\mathbf{H} = \frac{1}{3Z_f}\begin{bmatrix} 1 & 1 & 1 \\ 1 & h & h^2 \\ 1 & h^2 & h \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & h^2 & h \\ 1 & h & h^2 \end{bmatrix} \tag{2.67}$$

Hence:

$$\mathbf{Y}_{012} = \frac{1}{Z_f}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \mathbf{Y}_{abc} \tag{2.68}$$

Clearly, the zero sequence network is open-circuited, and the positive and negative sequence networks are connected as shown in Figure 2.8. The sequence currents are:

$$I_0 = 0$$
$$I_1 = -I_2 = \frac{E_1}{Z_1 + Z_2 + Z_F} \tag{2.69}$$

and the fault current is given by:

$$\begin{aligned} I_b &= I_0 + h^2 I_1 + hI_2 \\ &= \left(h^2 - h\right) I_1 \\ &= -j\sqrt{3} I_1 \end{aligned} \tag{2.70}$$

Hence, using Eq. (2.69):

$$\boxed{I_b = \frac{-j\sqrt{3}E_1}{Z_1 + Z_2 + Z_F}} \tag{2.71}$$

Line-to-Earth Fault

Let the fault (or single phase load) be between line "a" and earth (or neutral), and have an impedance Z_F :

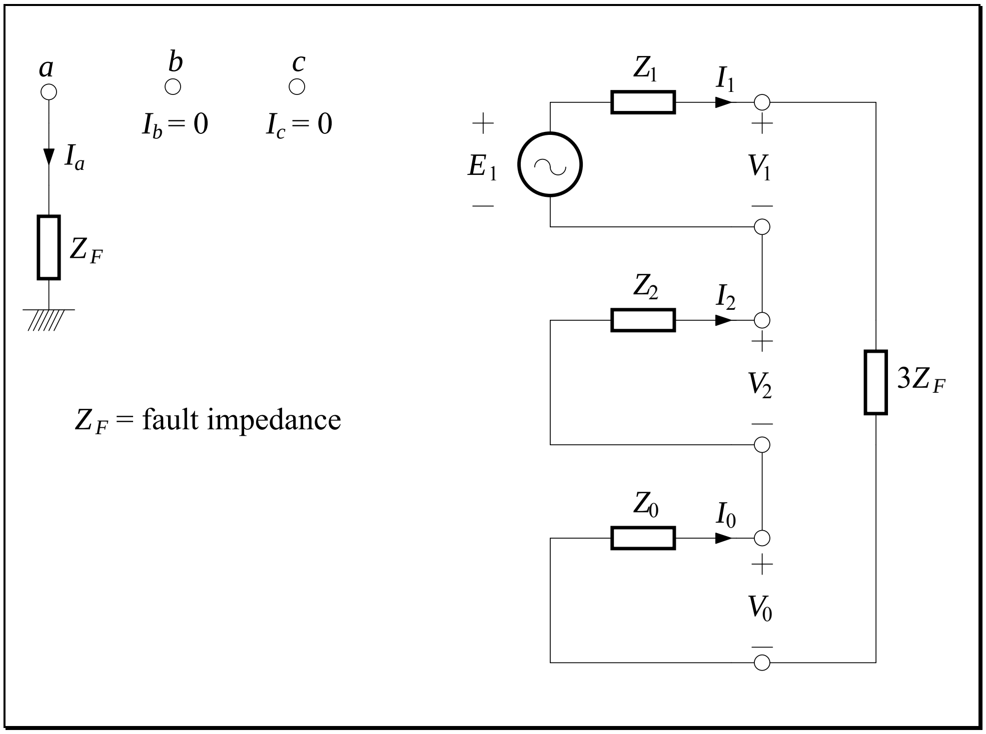


Figure 2.9

The fault admittance matrix is, by inspection:

$$\mathbf{Y}_{abc} = \frac{1}{Z_f}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.72}$$

Then:

$$\mathbf{Y}_{012} = \mathbf{H}^{-1}\mathbf{Y}_{abc}\mathbf{H} = \frac{1}{3Z_f}\begin{bmatrix} 1 & 1 & 1 \\ 1 & h & h^2 \\ 1 & h^2 & h \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & h^2 & h \\ 1 & h & h^2 \end{bmatrix} \tag{2.73}$$

Hence:

$$\mathbf{Y}_{012} = \frac{1}{3Z_f}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{2.74}$$

and:

$$\mathbf{I}_{012} = \mathbf{Y}_{012}\mathbf{V}_{012} = \frac{V_0 + V_1 + V_2}{3Z_f}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{2.75}$$

Clearly, the three sequence networks are connected in series as shown in Figure 2.9. The sequence currents are:

$$I_0 = I_1 = I_2 = \frac{E_1}{Z_0 + Z_1 + Z_2 + 3Z_F} \tag{2.76}$$

and the fault current is given by:

$$\boxed{I_a = I_0 + I_1 + I_2 = \frac{3E_1}{Z_0 + Z_1 + Z_2 + 3Z_F}} \tag{2.77}$$

Summary

- The per-unit value is defined as the ratio of a physical value of some quantity to a base value. They are used extensively in power systems analysis.

- Real power flow in an ideal "loss free" interconnector is determined only by the power angle (the phase difference between the end voltages).

- The average transferred reactive power in a "loss free" interconnector is determined only by the voltage magnitudes.

- The static stability limit in an interconnector is achieved when maximum real power transfer is achieved – this occurs when the phase difference between the end voltages matches the angle of the interconnector impedance.

- Various types of power system loads exhibit different sensitivities, with respect to relative power changes, to voltage and frequency variations.

- A set of unsymmetrical phasors can be expressed as the sum of three sets of symmetrical phasors, or *symmetrical components*, known as positive, negative and zero sequence components.

- Any three-phase network can be represented by an equivalent set of sequence networks, called the positive, negative and zero sequence networks.

- Any neutral-to-earth impedance only appears in the zero sequence, and is 3 times the original magnitude since the zero sequence currents for all phases are equal.

- Sequence networks are used to analyse unbalanced conditions in the system, such as faults (or a load unbalance). The sequence networks are connected so that the resulting network equations give the fault (or load) current and voltage.

References

Carmo, J.: *Power Circuit Theory Notes*, UTS, 1994.

Truupold, E.: *Power Circuit Theory Notes*, UTS, 1993.

Exercises

1.

A 3-phase transmission circuit has an impedance per phase of $5 + j35\,\Omega$. The load at the receiving end consumes 600 kW at unity p.f. and 13.2 kV (line voltage). Calculate sending end voltage magnitude, real, reactive and apparent power, using per-unit values, given that the 3-phase power base is 100 MVA, and the nominal line voltage is 220 kV.

Per-unit values of electrical quantities

2.

Two identical parallel connected 3-phase 6.6 kV 6.25 MVA generators feed into the LV winding of a 3-phase 6.6 kV / 66 kV transformer rated at 12.5 MVA. The HV winding of the transformer is connected to a 3-phase feeder. Find the total per-unit impedance of the circuit, as seen from the receiving end of the feeder, on a 12.5 MVA 66 kV base, given the following data:

Generator impedance (each) = (1 + j30) % based on ratings
Transformer impedance = (1 + j8) % based on ratings
Feeder impedance per phase = $10 + j14\ \Omega$

3.

A 3-phase 100 km transmission line has an impedance of $j80\,\Omega$ per phase. Resistance and capacitance can be neglected.

Power flow in an interconnector

(a) 100 MW is carried along the line from end 1 to end 2 while the voltages are maintained at 140 kV and 130 kV at ends 1 and 2 respectively. Calculate the power angle and the reactive power flow at each end.

(b) The line is operated at the static stability limit with voltages of 140 kV at each end. Calculate the complex power input and output of the line, and the voltage half-way along the transmission line.

4.

Modelling of power system loads

A composite load consists of heating, lighting and motors in equal proportions of real power. Estimate the percent change in real power resulting from a 5% fall in supply voltage.

5.

Symmetrical components - introduction

A generator, with its neutral connected to earth via a $10\,\Omega$ reactor, supplies the following line to earth voltages and line currents to an unbalanced load:

$$\begin{array}{lll} V_a = 18.0\angle 0°\text{ kV} & V_b = 13.3\angle -13.2°\text{ kV} & V_c = 12.0\angle 110°\text{ kV} \\ I_a = 200\angle 0°\text{ A} & I_b = 350\angle -140°\text{ A} & I_c = 400\angle 100°\text{ A} \end{array}$$

Find:

a) The symmetrical components of the above voltages.

b) The voltage to earth on the generator neutral.

c) The active power supplied (i) using phase coordinates
(ii) using sequence coordinates.

6.

A three-phase 50 Hz reactor consists of three coupled coils. Each coil has a self-inductance of $500\,\mu\text{H}$ and a resistance of $20\,\text{m}\Omega$. The mutual inductance between any two coils is $100\,\mu\text{H}$.

a) Calculate the sequence impedances of the reactor in complex ohms.

b) Express the results of a) as per-unit values, using a base voltage of 11 kV and a base power of 100 MVA.

7.

A generator is running on open circuit at a terminal voltage of 1.1 p.u. Assume generator impedances are:

$$Z_1 = Z_2 = j0.12\,\text{p.u.} \quad Z_0 = j0.06\,\text{p.u.}$$

The generator is equipped with a neutral reactor of such a value as to limit the line to earth fault current to 5.0 p.u.

Calculate the three line to earth voltages for a solid line to earth fault on “a” phase.

8.

A three-phase load consists of a $1\,\Omega$ resistor connected between terminals “a” and “b”, a $j\sqrt{3}\,\Omega$ reactor between terminals “a” and “c, and a $-j\sqrt{3}\,\Omega$ capacitor between terminals “b” and “c”. Using the matrix transformation in Eq. (2.50) prove that the load is balanced when the symmetrical applied voltage has the phase sequence *abc*, but unbalanced when the sequence is *acb*. (Compare with Exercise 1.12).