UNIVERSITY OF TECHNOLOGY SYDNEY

SCHOOL OF ELECTRICAL, MECHANICAL AND MECHATRONIC SYSTEMS

LECTURE NOTES

Transient Stability

SPRING SEMESTER, 2008

October 27, 2008

Transient Stability

Transient stability refers to the ability of a synchronous generator connected to the grid to maintain synchronism when subjected to severe disturbances due to heavy load changes, loss of generation in other units or faults in the system. When there is a sudden disturbance, the machine rotor starts to oscillate about its equilibrium position. These oscillations are modeled using swing equation.

1 Swing equation

When there is an unbalance between mechanical torque (T_m) and electrical torque (T_e), it will cause the rotor to accelerate or decelerate. The difference in the torque (T_a) is given by,

$$\begin{aligned} T_a &= T_m - T_e \\ J\frac{d\omega_m}{dt} &= T_m - T_e \qquad (1) \end{aligned}$$

where J is the moment of inertia of the rotor in kg.m^2 and ω_m is the angular velocity of the rotor in rad/s. Equation (1) can be normalised as follows:
Defining inertia constant H as

$$\begin{aligned} H &= \frac{1}{2}\ \frac{J{\omega_{0m}}^2}{VA_{base}} \\ J &= \frac{2HVA_{base}}{{\omega_{0m}}^2} = \frac{2H}{\omega_{0m}T_{base}} \qquad (2) \end{aligned}$$

where T_{base} = $\frac{VA_{base}}{\omega_{0m}}$. Combining (1) and (2) we get,

$$\begin{aligned} 2H\frac{d}{dt}\left(\frac{\omega_m}{\omega_{0m}}\right) &= \frac{T_m - T_e}{T_{base}} \\ 2H\frac{d\tilde{\omega}_r}{dt} &= \tilde{T}_m - \tilde{T}_e \qquad (3) \end{aligned}$$

where the variables superscripted with ˜ indicates per unit quantities and $\frac{\omega_m}{\omega_{0m}} = \frac{\omega_r}{\omega_0} = \tilde{\omega}_r$ and ω_r is the velocity of the rotor in electrical rad/s and ω_0 is its rated value.

If δ is the angular position of the rotor with respect to the rotating reference frame,

$$\delta = \omega_r t - \omega_0 t + \delta_0 \qquad (4)$$

where $\delta_0 = \delta$ at t = 0. Differentiating (4) twice with respect to time,

$$\begin{aligned} \frac{d\delta}{dt} &= \omega_r - \omega_0 = \Delta\omega_r \qquad (5) \\ \frac{d^2\delta}{dt^2} &= \frac{d\left(\Delta\omega_r\right)}{dt} = \frac{d}{dt}\left(\omega_0\tilde{\omega}_r - \omega_0\right) \qquad (6) \\ \frac{d^2\delta}{dt^2} &= \omega_0\frac{d\tilde{\omega}_r}{dt} \qquad (7) \end{aligned}$$

Substituting in (3),

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = \tilde{T}_m - \tilde{T}_e \tag{8}$$

To include the rotor damping effects, we include rotor damping constant K_D and proportional to change in speed and we have,

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = \tilde{T}_m - \tilde{T}_e - K_D\Delta\tilde{\omega}_r \tag{9}$$

Equation (9) is known as *swing equation*. In pu representation, torque and power are equal, so we can represent (9) as,

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = \tilde{P}_m - \tilde{P}_e - K_D\Delta\tilde{\omega}_r \tag{10}$$

2 Single machine on infinite bus (SMIB) model

Infinite bus is the one with constant frequency and voltage. Consider a generator G connected to infinite bus through a transformer and a transmission line as shown in Figure 1(a). The reactance of the line X_{line}, transformer X_{trans} and the transient reactance X'_d are indicated in Figure 1(b). E' is the generator voltage behind X'_d and E_{bus} is the infinite bus voltage. Generator voltage E' leads the E_{bus} by an angle δ. The given network can be reduced as shown in Figure 1(c) where $X_T = X'_d + X_{trans} + X_{line}$.

Let E' be the reference vector, the current I_t is given by,

$$\begin{aligned} I_t &= \frac{E'\angle 0 - E_{bus}\angle{-\delta}}{\jmath X_T} \\ &= \frac{E'\angle 0 - E_{bus}\left(\cos\delta - \jmath\sin\delta\right)}{\jmath X_T} \end{aligned} \tag{11}$$

Apparent power behind X'_d is given by,

$$\begin{aligned} S' &= P + \jmath Q = E' I_t^* \\ &= \frac{E' E_{bus}\sin\delta}{X_T} + \jmath\frac{E'\left(E' - E_{bus}\cos\delta\right)}{X_T} \end{aligned} \tag{12}$$

If we neglect the stator resistance loss, the airgap power will be the terminal power P_e.

$$\begin{aligned} P_e &= \frac{E' E_{bus}}{X_T}\sin\delta \\ &= P_{max}\sin\delta \end{aligned} \tag{13}$$

where $P_{max} = \frac{E' E_{bus}}{X_T}$. Equation (13) is diagrammatically represented in Figure 2. Under steady

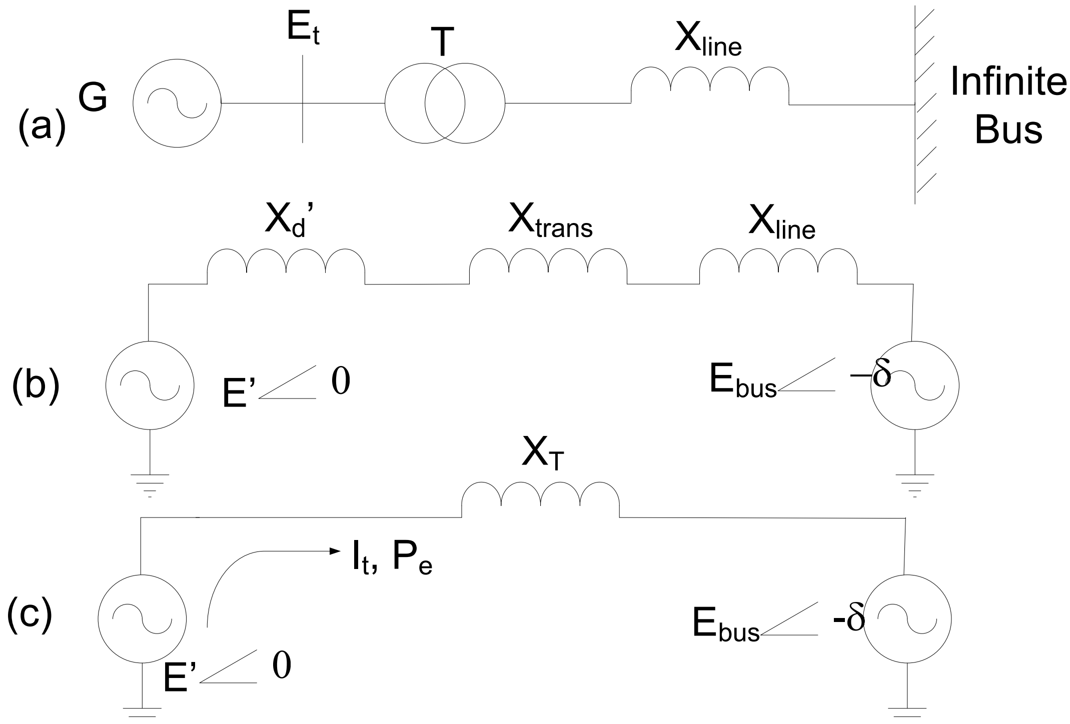

Figure 1: Single machine on infinite bus system

state P_m = P_e, and let the corresponding rotor angle be δ. From (13), P_{max} is inversely proportional to X_T. With X_T increased to X_{T1}, P_{max} reduces to P_{max1} as shown in Figure 2 and also the steady state angle δ increases to δ_1.

Combining (8) and (13) we get,

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta \tag{14}$$

2.1 Rotor angle response to sudden change in P_m

When the mechanical power P_m is increased to P_{m1} as shown in Figure 5, since $P_{m1} > P_e$, rotor accelerates increasing rotor angle towards δ_1. At y, $P_{m1} = P_e$, but the rotor speed is higher than the synchronous speed ω_0 and so the rotor angle continues to increase. This will increase P_e, now since $P_{m1} < P_e$, rotor will start decelerate. At z with δ_2, rotor speed reaches syn.speed but still $P_{m1} < P_e$ rotor continues to decelerate with speed falling below ω_0. The rotor retraces the path, after reaching x the rotor continues to oscillate about the new equilibrium point δ_1

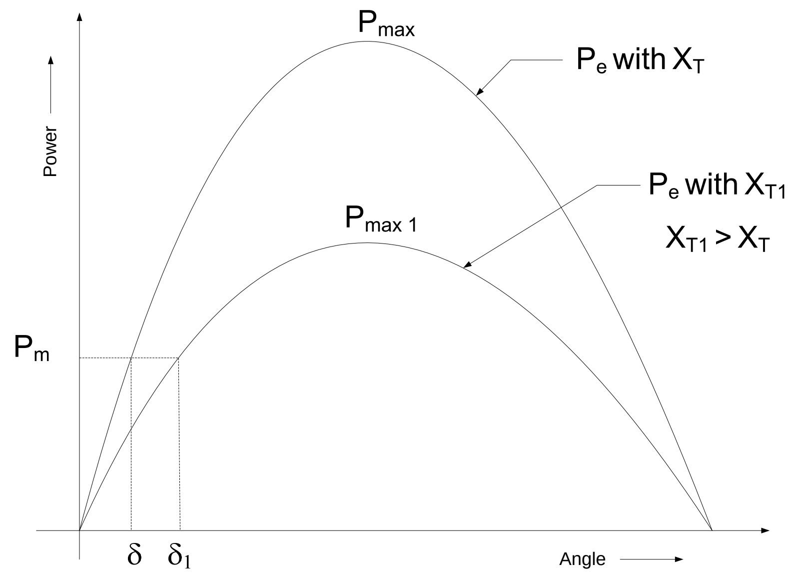

Figure 2: Power angle curve

2.2 Equivalent single machine system

We can represent a system consisting of many generators operating in parallel by an equivalent single machine system.
Find the equivalent single machine system of the following multimachine systems.

1. Each machine 30 kVA, H = 6 s, xd' = 0.25 pu. (Refer Figure 3).

2. G_1 = 30 kVA, H_1 = 6 s, xd_1' = 0.25 pu and G_2 = 60 kVA, H_2 = 4 s, xd_2' = 0.3 pu (Refer Figure 4).

3 Stability based on equal area criterion

Using SMIB model, the stability of rotor angle oscillations can be determined graphically without solving the swing equations. Even though this method cannot be applied to multimachine systems, it can provide basic understanding and information about critical clearing angle and stability limits.

From equation (14) we have

$$\frac{d^2\delta}{dt^2} = \frac{\omega_0}{2H}\ (P_m - P_e) \tag{15}$$

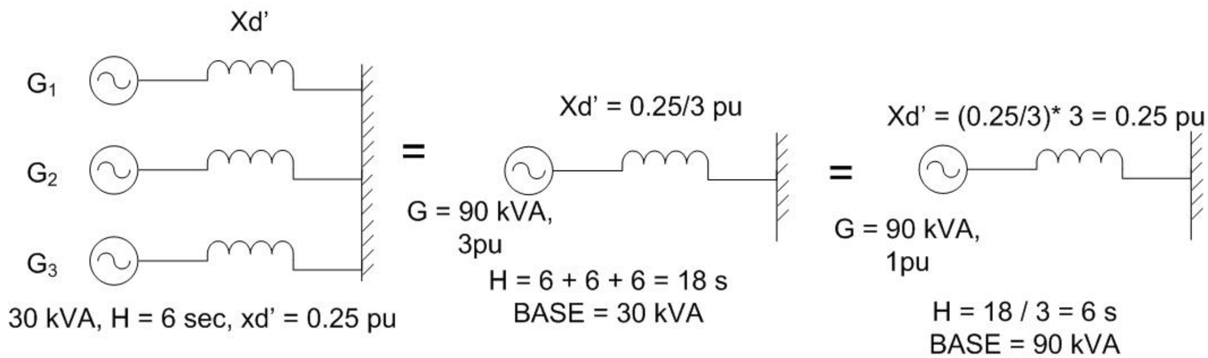

Figure 3: 3 M/C system

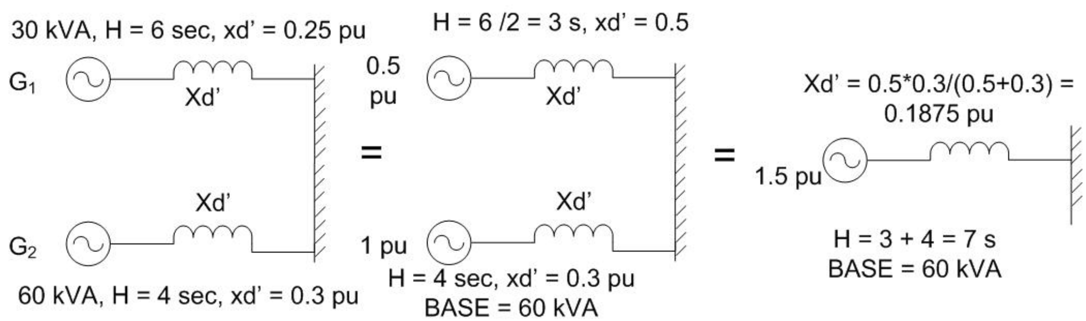

Figure 4: 2 M/C system

Multiplying by $2\frac{d\delta}{dt}$,

$$\begin{aligned} 2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} &= \frac{\omega_0}{2H}\ (P_m - P_e)\, 2\frac{d\delta}{dt} \\ \frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 &= \frac{\omega_0}{H}\ (P_m - P_e)\, \frac{d\delta}{dt} \end{aligned} \tag{16}$$

Integrating (16) we get,

$$\left(\frac{d\delta}{dt}\right)^2 = \int \frac{\omega_0}{H}\ (P_m - P_e)\, d\delta \tag{17}$$

Under steady state conditions $\frac{d\delta}{dt} = 0$, during disturbance δ will be changed, but after the removal of disturbance the system should regain steady state conditions without δ deviating unboundedly. For stability equation (17) should be equal to zero i.e,

$$\int_{\delta_0}^{\delta_2} \frac{\omega_0}{H}\ (P_m - P_e)\, d\delta = 0 \tag{18}$$

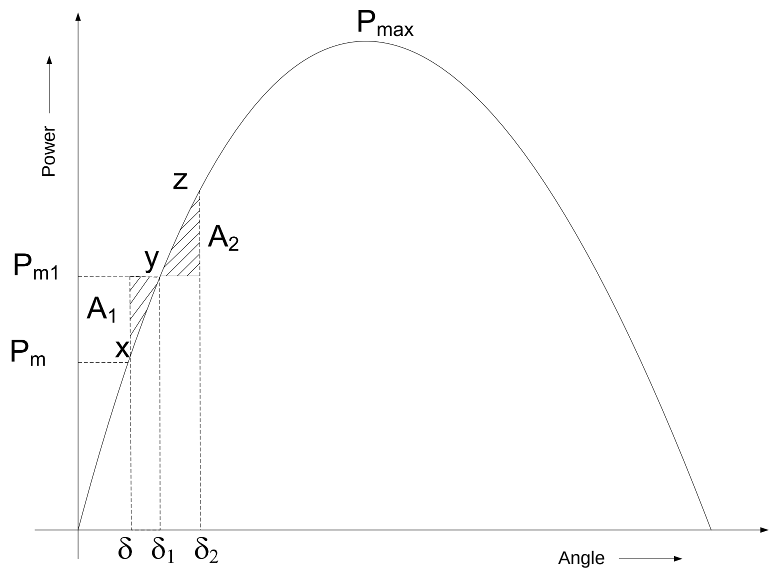

Figure 5: Rotor angle oscillations

where δ_0 is the steady state rotor angle, δ_2 is the maximum rotor angle (see Figure 5). For the equation (18) to be zero, the area described by the function $P_m - P_e$ must be zero. From Figure 5, the kinetic energy gained is given by area A_1 and is given by,

$$A_1 = \int_{\delta_0}^{\delta_1} (P_m - P_e)\, d\delta \tag{19}$$

The energy lost during deceleration is given by the area A_2 and is given by,

$$A_2 = \int_{\delta_1}^{\delta_2} (P_m - P_e)\, d\delta \tag{20}$$

for stability,

$$\text{Area } A_1 = \text{Area } A_2 \tag{21}$$
$$\text{i.e, kinetic energy gained} = \text{energy lost during deceleration}$$

Equation (21) represents the principle of stability through equal area criterion.

3.1 Stability during sudden input power change

For sudden change in input power stability can be found by using the stability criteria given by (21). Stability can be maintained only if we can find area A_2 in the power angle curve such that (21) is satisfied. The maximum rotor angle δ_{max} is given by the rotor angle corresponding to P_m for which $\frac{\pi}{2} < \delta > \pi$ as shown in Figure 6.

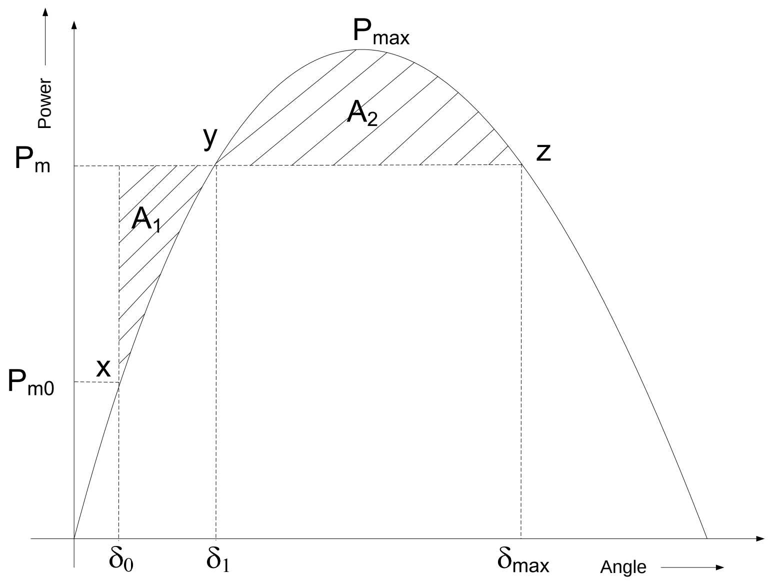

Figure 6: Power angle curve for sudden change in P_m

Using (13) and (21) we can write,

$$P_m(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{max} \sin \delta \, d\delta = \int_{\delta_1}^{\delta_{max}} P_{max} \sin \delta \, d\delta - P_m(\delta_{max} - \delta_1) \tag{22}$$

On simplification,

$$\begin{aligned} P_m(\delta_{max} - \delta_0) &= P_{max}\left[(-\cos\delta)_{\delta_0}^{\delta_1} + (-\cos\delta)_{\delta_1}^{\delta_{max}}\right] \\ &= P_{max}(\cos\delta_0 - \cos\delta_{max}) \end{aligned} \tag{23}$$

At stability limit $P_m = P_{max} \sin \delta_{max}$, substituting in (23),

$$\sin\delta_{max}(\delta_{max} - \delta_0) + \cos\delta_{max} = \cos\delta_0 \tag{24}$$

Equation (24) is a nonlinear equation which can be solved by iterative technique to get δ_{max}. With δ_{max} known we can find δ_1 and P_m as follows:

$$\delta_1 = \pi - \delta_{max} \quad \text{and} \quad P_m = P_{max}\sin\delta_1 \tag{25}$$

3.2 Stability during 3 phase fault

Fault at sending end. When fault occurs near the sending end as in Figure 7, no power is send to the infinite bus. If we neglect the resistance of transformer and generator P_e is zero during fault. Since P_e is zero, P_m is used to accelerate rotor and this increases rotor angle to δ_1. At

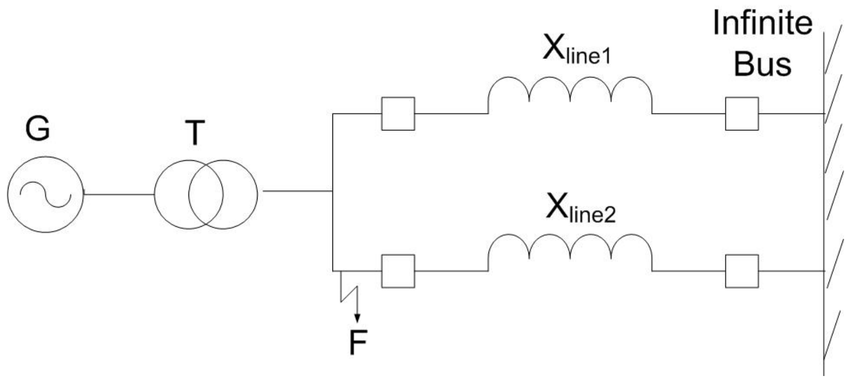

Figure 7: Fault at the sending end

δ_1, the fault is cleared and P_e takes the value corresponding to y as shown in Figure 8(a). At y since $P_e < P_m$, rotor decelerates to z and then returns back to x and starts to oscillate about δ_0. Critical clearing angle δ_c is reached when area A_2 becomes less than A_1 [see Figure 8(b)] and at z the rotor angle becomes δ_{max}.
Using (21)

$$\begin{aligned} P_m\left(\delta_c - \delta_0\right) &= \int_{\delta_c}^{\delta_{max}} \left(P_{max}\sin\delta - P_m\right)\, d\delta \\ &= P_{max}\left(\cos\delta_c - \cos\delta_{max}\right) - P_m\left(\delta_{max} - \delta_c\right) \\ P_m\left(\delta_{max} - \delta_0\right) &= P_{max}\cos\delta_c - P_{max}\cos\delta_{max} \end{aligned} \tag{26}$$

solving for δ_c,

$$\delta_c = \cos^{-1}\left[\cos\delta_{max} + \frac{P_m}{P_{max}}\left(\delta_{max} - \delta_0\right)\right] \tag{27}$$

To find the critical time t_c, substitute $P_e = 0$ in (15),

$$\frac{d^2\delta}{dt^2} = \frac{\omega_0}{2H}\, P_m \tag{28}$$

Integrating both sides from time 0 to t we get,

$$\begin{aligned} \frac{d\delta}{dt} &= \frac{\omega_0}{2H}\, P_m t \\ \delta &= \frac{\omega_0}{4H}\, P_m t^2 + \delta_0 \end{aligned} \tag{29}$$

at $\delta = \delta_{max}$, $t = t_c$, substituting in (29),

$$t_c = \sqrt{\frac{4H\left(\delta_{max} - \delta_0\right)}{\omega_0 P_m}} \tag{30}$$

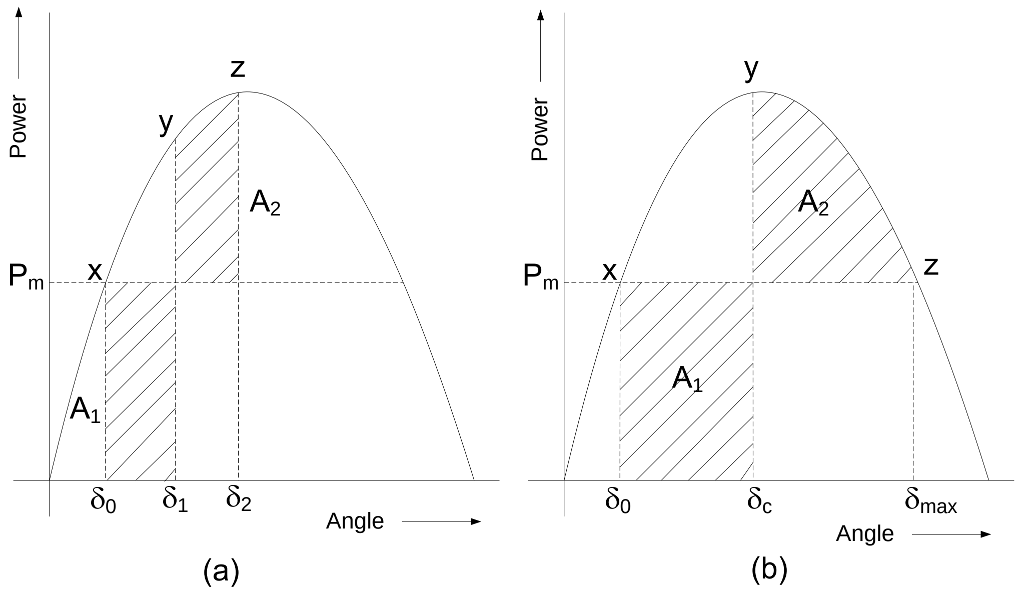

Figure 8: Power angle curve for fault at sending end

4 Numerical Example

Let us consider a 50 Hz single machine on infinite bus system. The machine is connected to infinite bus through a transformer and a transmission line. The system data is included in the Figure 9. A temporary 3 phase fault occurs at sending end and the system is restored back to normal after fault clearance. Find the critical clearing angle and critical clearing time above which the system will become unstable.

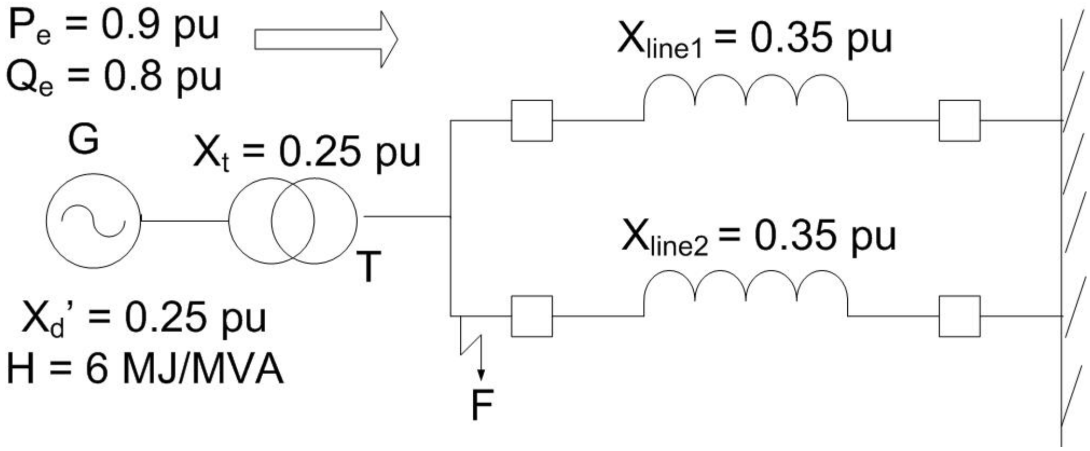

Figure 9: Power system for numerical example.

Generator Current $I_G = \dfrac{0.9 - \jmath 0.08}{1\angle 0} = 0.9 - \jmath 0.08\, pu$

Total reactance $X_{T1} = 0.25 + 0.25 + \dfrac{0.35}{2} = 0.6750\, pu$

Generator voltage behind $x'_d, E_q = V_{bus} - I_G X_{T1} = 1\angle 0 + \jmath 0.6750 \times (0.9 - \jmath 0.08)$

$= 1.2165\angle 29.9581\, pu$

Initial power angle $\delta_0 = \sin^{-1}\left(\dfrac{P_e X_{T1}}{E_q V_{bus}}\right) = 29.9581^0$

Maximum power angle $\delta_{max} = 180 - \delta_0 = 150.0419$

Critical clearing angle $\delta_c = \cos^{-1}\left[\cos\delta_{max} + \dfrac{P_m}{P_{max}}\left(\delta_{max} - \delta_0\right)\right] = 79.6180^0$

Critical clearing time $t_c = \sqrt{\dfrac{4H\left(\delta_{max} - \delta_0\right)}{\omega_0 P_m}} = 0.2712\, s$

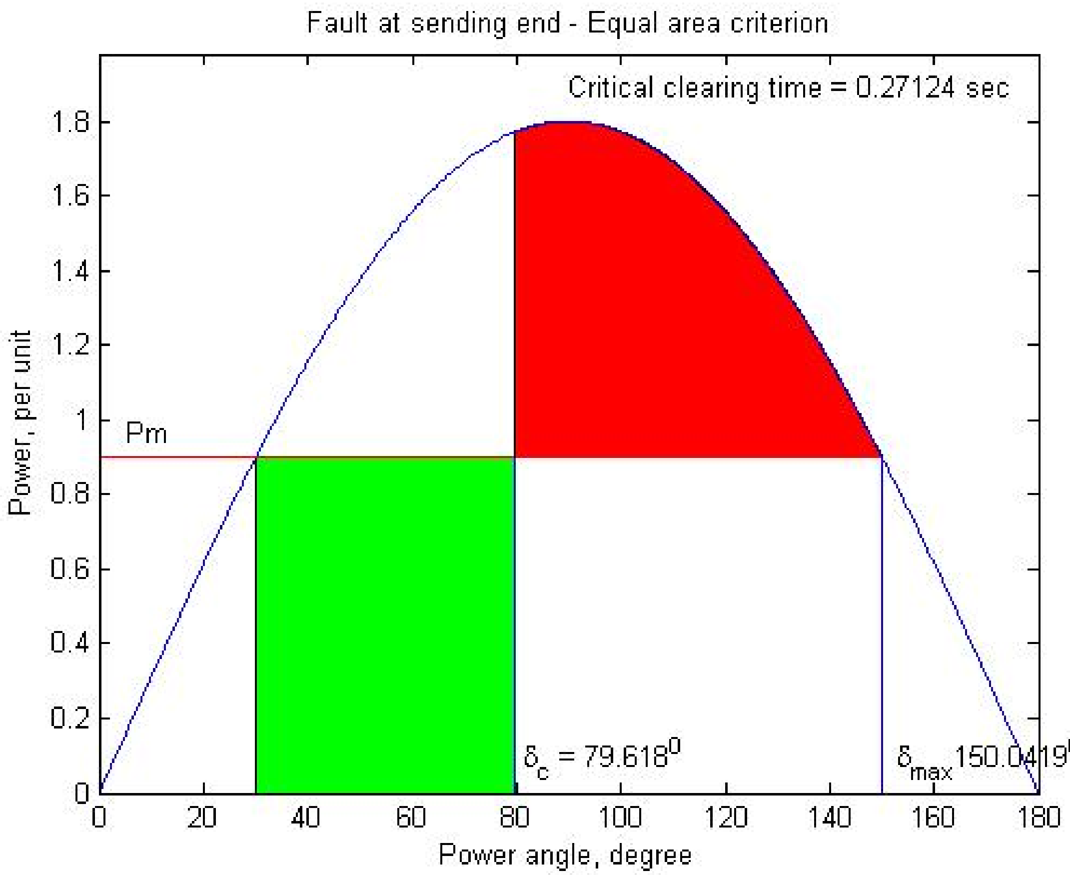

Figure 10: Power angle curve for the numerical example.